MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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PROFESSOR A. CAYLEY, F.R.S., Vice-President, in the Chair.

John Rand Capron, Esq., Guildown, Guildford, Surrey;
Dr. William Evans, Llanerchymedd, Anglesey;
Frank McClean, Esq., M.A., 23 Great George Street, Westminster; and
R. Meldola, Esq., F.C.S., 23 John Street, Bedford Row;

were balloted for and duly elected Fellows of the Society.

The Nebula in Orion—Request to Astronomers.

(*Communicated by E. B. Knobel, Esq.*)

I am engaged in observations of the central portions of the Nebula of *Orion*, and desire to compare all former drawings, both ancient and modern. I have in my possession the following published drawings of it:—

1656, Huyghens; 1673, Picard; 1731, Mairan; 1742, Long; 1758, Le Gentil (2); 1771, Messier; 1774, W. Herschel; 1779, Lefebvre; 1794, Schroeter; 1797–8, Schroeter (4); 1824, J. Herschel; 1837, J. Herschel; 1837, Lamont; 1839, De Vico; 1841, Rondoni (2); 1847, Lassell; 1848, W. C. Bond (2); 1854, Lassell; 1862, Liaponoff and Struve; 1862, Secchi; 1862, Tempel; 1865, G. P. Bond; 1867, Lord Rosse; 1868, Secchi; 1872, D'Arrest; 1874, Winlock and Trouvelot; 1875, Holden and Trouvelot.

Through the courtesy of the various authors, I have original (unpublished) sketches, or elaborate drawings, made by Cooper (?); Schmidt, 1861 (2); Lassell, 1862; and Trouvelot, 1875.

I shall be most grateful for notice of any drawing not mentioned in the above list, whether published or unpublished, or for a reference to unpublished observations of this object, particularly of the central nebulous portions.

Edward S. Holden.

United States Naval Observatory, Washington,
1877, *January* 31.

The Approaching Opposition of Mars—Request to Astronomers.

M. F. Terby, Doctor of Science, Louvain, Belgium, invites observers, particularly those in the equatoreal and southern regions, to make a large number of drawings of the planet *Mars* during the remarkable opposition of the present year, 1877. He recalls to them that he has published a Memoir, entitled "Aréographie, ou étude comparative des Observations faites sur l'aspect de la planète *Mars* depuis Fontana (1636) jusqu'à nos jours," t. xxxix. *Mem. Cour.* (4°) de l'Acad. R. de Belgique. This Memoir contains a large number of questions, which might be answered by the observers in the opposition of 1877; these refer to the principal doubtful points in Mr. Proctor's Chart. M. Terby begs the astronomers who have hitherto kindly communicated to him their observations to send him also their drawings of 1877; he hopes thus to perfect Mr. Proctor's already so excellent chart.

The Dunecht Observatory Publications.

Mr. Ralph Copeland writes to correct an error occurring in *Monthly Notices*, p. 199, where it is stated that the arrangement and reduction of vol. i. of the Dunecht Observatory Publications were performed by him. Although he is responsible for the general arrangement of the work, yet the whole of the computations were performed by Mr. Robert Copeland, of Manchester.

On Apparent Brightness as an Indication of Distance in Stellar Masses. By E. J. Stone, M.A., F.R.S., Her Majesty's Astronomer, Cape of Good Hope.

In the important investigations of the motion of the solar system in space by O. Struve and Airy, the distribution of the stellar distances has been made in accordance with the views of W. Struve. The correctness of these views has, from time to time,[a] been questioned. It may have been shown that some

astronomers have attached undue importance to the numerical accuracy of the results obtained by W. Struve; but I cannot consider that the average distribution of the distances of stars according to apparent brightness has been, or indeed ever will be, disproved. I do not know that there is much novelty in my views, but the subject presents itself to my mind in the following way. It is certain that the apparent brightness of a star will, *cæteris paribus*, vary inversely as the square of its distance from us; and an inspection of any table of annual parallaxes will prove that we have around us ample space for very considerable variations in the magnitudes of stars from the sole effects of distance.

The following table, extracted from Herschel's *Outlines of Astronomy*, will be sufficient for my present purpose:—

Star's name.	Annual parallax.
	″
α Centauri	0·976
61 Cygni	0·348
α Lyræ	0·155
Sirius	0·150
1830 Groombridge	(0·71) doubtful
ι Ursæ Majoris	0·133
Arcturus	0·127
Polaris	0·067
Capella	0·046

Without attaching undue importance to the numerical accuracy of the smaller values, we have here a proved range of stellar distances in at least the proportion of 1 to 10. It is certain, therefore, that our brightest stars might be changed to stars of about the tenth magnitude by a mere shift of our relative position in space. But there are not the slightest grounds for our supposing that the relative distance of the stars from us is restricted to any such proportions as 1 to 10. Such a proportion merely marks the limit beyond which we cannot discriminate with much certainty between the distances of the stars. Annual parallaxes of less than a tenth of a second of arc are sensibly equal, because the limit of our present power of accurate measurement has been nearly reached. If we assume, therefore, that the magnitudes of the stars depend chiefly upon their distances from us, our assumption rests upon a *vera causa*, and we have independent proof of the sufficiency of the assumed cause to produce the observed differences in stellar magnitudes.

But I think it may be shown that such an assumption is not a mere barren hypothesis. I know of no more striking fact in Astronomy than the rapid increase in the number of the stars as we proceed down the scale of magnitudes. Every increase of

optical power appears only to bring this fact into greater prominence.

If we accept distance as the principal factor in determining stellar magnitudes, the rapid increase in the number of the fainter stars not only becomes intelligible, but is a necessary consequence of our assumption on the most general distribution of stellar masses which we can conceive.

Suppose that there are n different degrees of intrinsic brightness in the stars around us. Suppose, for the purposes of numerical illustration, that we have over any surface these classes of stars distributed in any definite proportions. Since the brightness of a star is classed under discontinuous magnitudes, we can logically suppose, for our present purpose, that the stellar distribution is discontinuous also over the surfaces of small portions of spheres whose radii are such that a star shifted from one surface to the next would be changed one magnitude. With such a distribution of stars, without any assumptions of uniform distribution in all directions or at all distances, and with any proportions amongst the stars of different intrinsic brightness which could be admitted as probable, the large increase in the number of the fainter stars becomes a necessary consequence of the supposed distribution. I shall, however, for simplicity, introduce the conditions that the stars of different intrinsic brightness are to be supposed equally distributed, and their number in any given direction proportional to the area over which they are distributed. Such conditions are certainly not strictly true, and so far as they are untrue their tendency will be to make the results of the computations based upon them differ from the results of observation; but on the large average these conditions are probably close approximations to the truth. Through our position conceive a cone of small solid angle to be drawn in any direction. Let $x_1, x_2 \ldots x_n$ be the radii of the spheres which cut the cone at distances such that a star would be changed one magnitude by being shifted from one of the surfaces to the consecutive surfaces; x_1 being such a distance that the stars of the first degree of intrinsic brightness will at that distance appear stars of the first magnitude. Then, if $ds_1, ds_2, \ldots ds_n$ are the portions of the spheres intercepted by the cone, the number of the stars of the different magnitudes in the direction under consideration will be—

1st magnitude number = C (some constant).

2nd " " $= C\left(1 + \frac{ds_2}{ds_1}\right)$.

3rd " " $= C\left(1 + \frac{ds_2}{ds_1} + \frac{ds_3}{ds_1}\right)$.

nth " " $= C\left(1 + \frac{ds_2}{ds_1} + \frac{ds_3}{ds_1} + \frac{ds_4}{ds_1} + \ldots + \frac{ds_n}{ds_1}\right)$;

for stars of the first degree of intrinsic brightness only appear of the first magnitude when found on ds_1; of the second magnitude on ds_2; and of the nth on ds_n.

Stars of the second degree of intrinsic brightness appear of the second magnitude on ds_1; of the third on ds_2; and of the $(n+1)$th on ds_n. Stars of the third degree of intrinsic brightness appear of the third magnitude on ds_1; of the fourth magnitude on ds_2, &c.

Consequently, therefore, since a similar result holds in all directions, we have generally the number of nth magnitude stars

$$= A\left(1 + \frac{ds_2}{ds_1} + \frac{ds_3}{ds_1} + \ldots + \frac{ds_n}{ds_1}\right)$$

$$= A\left(1 + \left|\frac{x_2}{x_1}\right|^2 + \left|\frac{x_3}{x_1}\right|^2 + \ldots + \left|\frac{x_n}{x_1}\right|^2\right).$$

It remains to determine the numerical values of the ratios involved.

In Sir J. Herschel's volume of Cape Observations will be found some experiments showing that, if the double star α *Centauri* be taken as the standard, the brightness of stars of the first four magnitudes of the ordinary scale are very approximately

$$= \frac{1}{(1{\cdot}414)^2} \text{ for 1st magnitude stars.}$$

$$= \frac{1}{(2{\cdot}414)^2} \text{ for 2nd } \quad ,, \quad ,,$$

$$= \frac{1}{(3{\cdot}414)^2} \text{ for 3rd } \quad ,, \quad ,,$$

$$= \frac{1}{(4{\cdot}414)^2} \text{ for 4th } \quad ,, \quad ,,$$

Such a scale will not, however, be true for the fainter stars.

In the introduction to Carrington's *Redhill Catalogue of Stars*, there are some results of experiments with varying apertures, showing that from the fifth to the ninth magnitude of the ordinary scale the brightness of each magnitude is fairly represented by the relation

Brightness of nth magnitude $= 2{\cdot}75 \times$ that of $(n+1)$th.

I shall exhibit the proportional number of the stars of different magnitudes when the ratios of $\frac{x_n}{x_1}$ are determined with both these light ratios. We must expect, if the supposed distribution is any close approximation to the truth, to find that the computed numbers will agree with the observed numbers, for the first four magnitudes better with Herschel's scale, and for the last five magnitudes better with Carrington's scale.

With Herschel's scale

$$\left|\frac{\overline{x_n}}{x_1}\right|^2 = \left(\frac{n\cdot 414}{1\cdot 414}\right)^2;$$

And the number of nth magnitude stars

$$s_n = \frac{CA}{(1\cdot 414)^2}\{(1\cdot 414)^2 + (2\cdot 414)^2 + \cdots + n\cdot 414)^2\};$$

$$s_n = C\left(\frac{n^3}{3} + 0\cdot 914\, n^2 + 0\cdot 752\, n\right).$$

Next, assuming Carrington's light ratio, we have

$$s_n = C\{1 + 2\cdot 75 + (2\cdot 75)^2 + \cdots + (2\cdot 75)^{n-1}\};$$

$$s_n = C\frac{(2\cdot 75)^n - 1}{1\cdot 75}.$$

The following Table exhibits certain numbers attributed to Argelander and the results of the two formulæ. The number of first magnitude stars is made equal for the purposes of comparison :—

Mag.	Argelander's numbers.	Computed numbers Herschel's scale.	Computed numbers Carrington's scale.	Mean of Compd.
1	20	20	20	20
2	65	78	75	76
3	190	195	226	210
4	425	390	642	516
5	1,100	683	1,786	1,234
6	3,200	1,094	4,931	3,012
7	13,000	...	13,581	...
8	40,000	...	37,360	...
9	142,000	...	102,780	...

It will be seen that the numbers based on Herschel's scale do agree best for the first four magnitudes, and those based on Carrington's scale best for the fainter magnitudes; whilst a mean between the two agrees best with the observed numbers for the end of Herschel's and the commencement of Carrington's scale. This is what might have been expected. The computed number of stars of the ninth magnitude falls short of the observed number by forty thousand; but when it is considered that this discrepancy is not greater than would arise from an uncertainty of estimation of about a tenth of a magnitude at the end of the scale, the agreement between the computed and observed numbers will, I think, be considered sufficiently close to prove that the supposed distribution of the stars must be a somewhat close approximation to the truth.

If the distribution which I have supposed be any close

approximation to the truth, the average distances of the fainter stars must be greater than those of the brighter stars; but it by no means follows that any one faint star is further from us than any one much brighter star. In fact, the supposition made has been that stars of all magnitudes may exist at all stellar distances.

On Observations of Contacts of the Limb of Venus, or Mercury, with that of the Sun. By Prof. Simon Newcomb.

During the past four years, I have had occasion to give some attention to the problem of obtaining accurate observations of the various phases of contact between the limb of an interior planet and that of the Sun. My views, as now presented, have been derived from the study of an artificial representation of a transit of *Venus*, aided by general considerations on the subject, and by the opinions and investigations of others. I am induced to present them at the present time by the researches of M. André, and the papers of Mr. Stone and Father Perry in the *Monthly Notices* for December last.

(1) If I might be allowed to criticise certain of the views of others in a general way, I should say that one defect of much of the reasoning on this subject is this: it has too generally been assumed that the geometric outlines of *Venus* and the Sun, considered as mathematical lines, can be noted in observation with the same sort of definiteness and precision as that with which the mind conceives them; and sufficient attention has not been paid to the practical difficulties which the eye meets with in representing this geometric conception. I conceive that the question whether a certain phase can or cannot be definitely distinguished and observed by the eye is to be settled by actual trial, and by a consideration of the imperfections of vision, rather than by a consideration of its purely geometric definiteness of form.

(2) One result of the trials with the artificial transit is, that there is a certain phase near that of external contact which can be observed with the same order of precision as the internal contact, provided that the proper conditions are fulfilled. Among these conditions are, that the observer shall previously have practised on the artificial transit; that he shall know at exactly what point of the Sun's limb to look for the first contact; that he shall know when to look for the contact with an uncertainty of not much less than half a minute, nor much more than a minute; and that he shall have a telescope of fixed size and power. Of course the phase thus observed will not be geometric contact, but that occurring at the time when the notch in the Sun's limb first becomes visible. This phase varies much less with variations in the atmospheric condition and in the size and power of the telescopes than might have been supposed.

(3) The phase of external contact at egress is more uncertain than at ingress, owing to the doubt of the observer as to whether the notch has or has not disappeared from his view; that doubt extending over a longer period than the doubt as to when he first sees the notch at ingress.

(4) Referring now to the spectroscopic observation of the external contact, and to Father Perry's suggestion for rendering the projection of the planet on the chromosphere visible by means of a coloured glass in order to observe this contact, which two methods are the same with respect to the optic phenomena designed to be produced, I do not think it possible to obtain a precise observation in either of these ways. The phenomenon to be observed would then be the contact of a dark limb (that of the planet) with an adjacent bright limb (that of the Sun), on a background intermediate in tint (that of the chromosphere). The result must then depend very largely on the amount of irradiation, and on the observer's acuteness of vision in seeing the slowly diminishing line of chromosphere between the planet and the Sun. The planet will, in reality, be invisible: all the observer can see will be the outline of the chromosphere, as cut out by the planet; and when the latter gets so near the limb of the Sun that the chromosphere itself is invisible—owing to the overpowering brightness of the Sun's limb—the outline of the planet can no longer be traced, and will not reappear until it is seen projected on the background of the Sun itself. I conceive, therefore, that any attempt to make different observers certainly note the same phase of such a phenomenon will result in failure. It would, however, be of interest to try the method artificially.

(5) In describing the phenomena near the time of internal contact, I shall consider the planet to be approaching egress. For reasons which I shall soon mention, the artificial transit was placed at a distance of about eleven hundred yards from the place of observation, so that a greater or less amount of atmospheric undulation was always present. Supposing, then, that the planet was approaching the Sun's limb, and the thread of light growing thin, owing to the approach of contact, the first thing which an observer would remark was that, in consequence of the constant changes of outline caused by atmospheric disturbance of the images, no set of phenomena could be described as invariable. It would be necessary to combine judgment with sight, by considering what might be called a *mean* phenomenon. Different observers might form different judgments as to what this mean phenomenon was. It would, however, always be seen that as the thread of light became quite thin, it looked darker than the rest of the Sun; and unless the atmosphere was more steady than is usual in the day-time, the line of light would occasionally be broken up into two or more very irregular threads or twists of light, which would gradually grow fainter until they would occasionally almost disappear from view. Repeated trials showed that the time of true internal contact was marked by the moment at which light entirely ceased to glimmer across the dark space

formed by the approach of the limb of the planet to that of the Sun.

(6) Every attempt to estimate an apparent contact, or moment at which the limbs of *Venus* and the Sun were tangent to each other, without reference to the appearance of the thread of light, was a total failure. It was, in fact, impossible to make any such estimate without an uncertainty of half a minute or more. This will not appear surprising if we reflect that the outlines of *Venus* and the Sun cannot present themselves to the vision as geometric lines, but only as more or less indefinite edges of a visible surface of sunlight; which visible surface disappears most gradually near the region of contact, and, in fact, at the moment of ideal "apparent contact" cannot be seen at all at the point of contact. Anyone who wishes to satisfy himself on this point has only to examine a series of figures in which the black drop is represented, and try to decide which of them represents apparent contact. If he wishes to come as near nature as possible, he must shade off the outlines both of *Venus* and the Sun, so that they shall terminate in a soft border, and view the picture through a rising current of hot air.

(7) Any artificial representation of these phenomena, in which the bright surface of the Sun is surrounded by a dark background, must fail to be correct in a very important particular. As a matter of fact, we know that the atmosphere immediately around the Sun's limb is of dazzling brilliancy. In meridian observations of the Sun, when the light is so cut down by a dark glass as no longer to dazzle the eye, the five spider-lines are visible on the background of the atmosphere; else it would be impossible to observe the transit of the Sun's first limb. This brilliancy of the atmosphere must greatly diminish irradiation; and if the Sun is observed through haze (as must often be the case in observations of contact when the Sun is near the horizon), irradiation may be entirely destroyed. It will therefore be impossible to observe an actual internal contact of *Venus* or *Mercury* with a precision corresponding to that of an artificial contact on a black background.

(8) The atmosphere affects the phenomena of contact in three ways:—

α. By illuminating the background as just described.

β. By producing undulations in the outlines of the images, thereby preventing the phenomena from being invariable.

γ. By softening the outlines of the Sun's limb, and thus rendering it more or less indefinite.

The artificial transit to which I have alluded was placed at a distance, in order that all these effects might be produced and studied; otherwise, the optic phenomena of contact may be entirely different. For instance, because the black drop is sharply seen in the artificial transit when the background is quite black, it does not follow that it will be noticed in the actual transit.

(9) A good idea of many of the causes of uncertainty to

which I have alluded may be formed by examining a series of photographs of internal contact taken by the Janssen process. One will immediately appreciate the distinction on which I have insisted, between geometric lines on the one hand and the outlines of *Venus* and the Sun on the other.

(10) I have said little of the black drop; partly because I do not think there can be much room for a real difference of opinion respecting its nature and causes, and partly because the question whether it is seen is of entirely secondary importance, except as affording some indication of the sharpness of definition. In looking at the artificial transit, it was very easy, about the moment of internal contact, by taking a general mean outline of the undulating image of the planet, and imagining that outline continued across the undulating line of light and darkness mixed, into which the ideal thread was reduced by the imperfections of vision, to see something like a black drop. On the other hand, another observer, with his attention fixed solely on the thread of light, would see nothing of the sort. The final disappearance of the thread of sunlight being the only phase which can be actually observed, an observer fixing his attention exclusively on this would not see any black drop at all, unless the amount of irradiation was exceptionally great.

(11) I see little or nothing in Mr. Stone's views radically inconsistent with those which I have here set forth. His remark on his observation of the egress of *Venus* in 1874, "the apparent contact was rendered more difficult than would otherwise have been the case by the appearance of this ligament, or, in other words, by the disappearance of a sensible portion of the Sun's limb near the point of contact," is in entire accord with the views I have expressed in paragraph 6.

(12) The general conclusion to which I am led is that there is but one phase of each contact which can be observed with any approach to accuracy, namely:—

α. The time when the notch made by *Venus* advancing on the Sun becomes visible.

β. The time at which true sunlight is first seen all the way around the following limb of the planet.

γ. The planet approaching egress, the time when it first completely cuts off the true limb of the Sun, and the space connecting the limb of the planet with the sky becomes as dark as the planet itself.

δ. The time when the last limb of the planet, leaving the limb of the Sun, disappears from view.

If an observer, at the time of internal contact, does not note, or try to note, the phases β and γ, there is no definite phase to which his observation can be referred. The old notion that at second internal contact there is a sudden formation of the black drop, which marks the moment of true contact, has been so completely exploded that it needs no further refutation. The more clear and dark the atmosphere around the Sun, the more rapidly

will contact appear to form, whether a black drop is seen or not; but under no circumstances under which an actual transit is likely to be observed for parallax will it be really sudden. When it appears so it is only because the observer failed to notice the gradual darkening and breaking up of the thread of light. From the commencement of this darkening and diffusion, until the "apparent contact," which comes last of all, there are a series of progressive changes, which may extend over a period ranging anywhere from twenty or thirty to ninety seconds, at any point of which a random observation of internal contact may fall. The worse the definition, and the lower the planet, the greater the range; but the time of true contact is always near the mean of the period.

Washington,
1877, *February* 22.

Ephemeris for Physical Observations of Jupiter 1877.
By Mr. A. Marth.

Greenwich Noon. 1877.	Angle of position of ♃'s Axis.	Longitude of ♃'s Meridian directed to the Earth.	diff.	Latitude of Earth above ♃'s Equator.	Latitude of Sun above ♃'s Equator.	Annual Parallax.		Equatoreal Diameter.
	°	°	°	°	°	°	°	″
Mar. 31	357·72	104·8		−2·31	−2·45	−10·81		38·71
			4353·1				+ ·14	
Apr. 5	357·59	137·9		2·31	2·43	10·67		39·33
			3·2				·21	
10	357·51	171·1		2·30	2·42	10·46		39·96
			3·3				·29	
15	357·46	204·4		2·30	2·41	10·17		40·59
			3·3				·37	
20	357·44	237·7		2·29	2·39	9·80		41·22
			3·4				·45	
25	357·46	271·1		2·29	2·38	9·35		41·85
			3·5				·53	
30	357·52	304·6		2·29	2·37	8·82		42·46
			3·5				·61	
May 5	357·61	338·1		−2·28	−2·35	−8·21		43·04
			3·6				·67	
10	357·74	11·7		2·28	2·34	7·54		43·60
			3·6				·75	
15	357·90	45·3		2·28	2·32	6·79		44·13
			3·6				·82	
20	358·09	78·9		2·28	2·31	5·97		44·60
			3·7				·87	
25	358·31	112·6		2·28	2·30	5·10		45·03
			3·7				·93	
30	358·55	146·3		2·28	2·28	4·17		45·40
			3·7				·97	
June 4	358·82	180·0		−2·28	−2·27	−3·20		45·69
			3·7				1·01	
9	359·10	213·7		2·28	2·25	2·19		45·92
			3·7				1·03	

Greenwich Noon. 1877.	Angle of position of ♃'s Axis.	Longitude of ♃'s Meridian directed to the Earth.	diff.	Latitude of Earth above ♃'s Equator.	Latitude of Sun above ♃'s Equator.	Annual Parallax.		Equatoreal Diameter.
	°	°	°	°	°	°		″
June 14	359·39	247·4		2·28	2·24	1·16		46 07
			3·7				1·04	
19	359·68	281·1		2·28	2·22	−0·12		46·14
			3·6				1·05	
24	359·98	314·7		2·28	2·21	+0·93		46·13
			3·6				1·04	
29	0·28	348·3		2·27	2·19	1·97		46·05
			3·5				1·01	
July 4	0·56	21·8		−2·27	−2·18	+2·98		45·88
			3·5				·99	
9	0·83	55·3		2·26	2·16	3·97		45·64
			3·4				·95	
14	1·08	88·7		2·26	2·15	4·92		45·33
			3·3				·89	
19	1·31	122·0		2·25	2·13	5·81		44·95
			3·2				·84	
24	1·51	155·2		2·24	2·12	6·65		44·52
			3·1				·77	
29	1·68	188·3		2·23	2·10	7·42		44·04
			3·1				·71	
Aug. 3	1·82	221·4		−2·22	−2·08	+8·13		43·53
			2·9				·64	
8	1·93	254·3		2·21	2·07	8·77		42·97
			2·8				·56	
13	2·00	287·1		2·20	2·05	9·33		42·40
			2·8				·48	
18	2·03	319·9		2·19	2·03	9·81		41·80
			2·6				·40	
23	2·03	352·5		2·18	2·02	10·21		41·20
			2·6				·33	
28	1·99	25·1		2·17	2·00	10·54		40·59
			2·5				·25	
Sept. 2	1·91	57·6		−2·15	−1·98	+10·79		39·98
			2·4				·17	
7	1·80	90·0		2·14	1·97	10·96		39·38
			2·3				·10	
12	1·66	122·3		2·12	1·95	11·06		38·78
			2·2				+ ·03	
17	1·48	154·5		2·11	1·93	11·09		38·20
			2·2				− ·04	
22	1·27	186·7		2·09	1·92	11·05		37·64
			2·1				·11	
27	1·03	218·8		2·07	1·90	10·94		37·10
			2·0				·17	
Oct. 2	0·76	250·8		−2·06	−1·88	+10·77		36·58
			2·0				·22	
7	0·46	282·8		2·04	1·86	10·55		36·08
			1·9				·28	
12	0·13	314·7		2·02	1·85	10·27		35·60
			4351·9				− ·34	
17	359·78	346·6		2·00	−1·83	9·93		35·15

Assumed daily rate of rotation 870°·60. The "annual parallax" is the difference of the Jovicentric longitudes of the Sun and Earth, reckoned in the plane of Jupiter's equator.

The inclinations γ and the ascending nodes Γ of the orbits of the satellites in reference to the plane of *Jupiter's* equator are the following, the nodes being reckoned from the point of the vernal equinox of the planet:—

1877.	Sat. I.		Sat. II.		Sat. III.		Sat. IV.	
	γ_1	Γ_1	γ_2	Γ_2	γ_3	Γ_3	γ_4	Γ_4
	°	°	°	°	°	°	°	°
Mar. 1	0·0098	40·9	0·4510	49·52	0·1990	274·46	0·3298	330·54
Apr. 30	98	39·5	·4518	47·51	·1981	273·91	·3290	330·53
June 29	98	37·9	·4525	45·49	·1972	273·35	·3281	330·50
Aug. 28	98	36·2	·4532	43·47	·1964	272·80	·3274	330·45
Oct. 27	0·0098	34·5	0·4539	41·45	0·1956	272·24	0·3267	330·39

On Two Ancient Occultations of Planets by the Moon, observed by the Chinese. By J. R. Hind, F.R.S.

In the Appendix to the *Connaissance des Temps* for 1810 will be found a considerable number of observations taken from the Chinese astronomical records by the Jesuit missionary Gaubil, whose manuscript relative to the Chinese observations of comets was so much used by Pingré in the preparation of his *Cometographie*. Amongst the numerous occultations comprised in this list, there are two, and two only, in which the hour, or rather the particular two-hourly interval, at which the occultation took place is distinctly stated, and it is to these occultations that the present Note refers.

I have calculated the circumstances of these phenomena with the view to check a system of lunar motions previously applied in searching for the dates of several solar eclipses recorded on the Nineveh tablets, to which my attention had been directed by Mr. J. W. Bosanquet, and which, as compared with several historical total eclipses, of which the dates are well known, appeared to require but small modification. I have employed Damoiseau's Tables of 1824, adapting his main arguments to the results for secular motions &c. obtained by the Astronomer Royal from the discussion of the Greenwich lunar observations from 1750 to 1830, retaining the last values given by Hansen for terms depending on the square of centuries, which I had found to fit in best with the Greenwich motion proportional to time, at least as regards the secular acceleration. For the Sun and planets Le Verrier's Tables were adopted.

The earliest occultation recorded by Gaubil is one of the planet *Mars* by the Moon, and is thus translated:—

"Dynastie des Han occidentaux, la cour à Si-gan-fou du Chen-sy. An 69 (B.C) = 1re année *Ti-tsie*, 1re Lune, jour *vou-ou* (14 février) à la 2e veille de 9 à 11 heures du soir, la Lune éclipsa *Mars*, entre les constellations *Kio* et *Kang*."

B.C. 69, February 14.

Paris M.T. h	Sun's True Longitude. ° ′ ″	Log. R.		
2	322 52 19·9	9·9979038	Sideral time at 0 h	21 h 23 m 31 s
6	323 2 16·9	9·9979208	Equation of time (Subtractive from Mean Time).	17 13

h	Mars' Helioc. Longitude. ° ′ ″	Mars' Helioc. Latitude. ° ′ ″	Mars' log. Rad. Vect.
2	166 58 50·5	+1 20 50·0	0·2062127
6	167 2 44·4	+1 20 44·7	0·2061696

h	Mars' Apparent R.A. ° ′ ″	Mars' Apparent Decl. ° ′ ″	
2	196 49 44	−4 20 44	Log. distance 9·90774.
6	196 49 51	−4 20 38	

h	Moon's R.A. ° ′ ″	Moon's Decl. ° ′ ″		
5	196 49 27	−3 46 31	Hor. Parallax	60 8·5.
6	197 23 58	−3 57 50	Semi-diameter	16 23.

I take the position of Si-gan-fou from M. Biot's *Dictionnaire des Noms Anciens et Modernes des Villes et Arrondissements de l'Empire Chinois*—

Long. 106° 37′ 45″ E. of Paris; Lat. 34° 16′ 45″;

and from the above data find a visible occultation :—

	h	m	
Immersion February 14 at	10	22	Apparent time at Si-gan-fou.
Emersion ,, ,,	10	59	

The immersion is therefore made to occur well within the two-hourly interval named in the Chinese annals, and the Moon was in *Virgo*, between the constellations *Kio* and *Kang*.

The second occultation to which I have referred is one of the planet *Venus*, thus given by Gaubil :—

"Dynastie des Tsin orientaux, la cour à Nanking, appelé alors Kien-Kang. An 361=5^{e} année, 1re Lune, jour *y-tcheou* (20 *Mars*) à l'heure *tchin* (temps de 7 à 9 heures du matin), la Lune éclipsa *Vénus* dans la constellation *Goey*."

Calculating in the same manner as before, I find the following figures :—

A.D. 361, March 19.

Paris M.T. h	Sun's True Longitude. ° ′ ″	Log. R.		
12	359 47 39·6	0·0018164	Sideral time at 0 h	23 h 49 m 28 s
13	359 50 6·2	0·0018113	Equation of Time (Subtractive from Mean Time).	7 49

h	Venus' Helioc. Longitude. ° ′ ″	Venus' Helioc. Latitude. ° ′ ″	Venus' Log. Rad. Vect.
12	239 0 56·6	+0 9 55·5	9·8614762
13	239 4 54·3	+0 9 41·0	9·8614777

h	Venus' Apparent R.A. ° ′ ″	Venus' Apparent Decl. ° ′ ″	
12	317 36 26	−16 18 37	Log. distance 9·94883
13	317 39 14	−16 17 59	

h	Moon's R.A. ° ′ ″	Moon's Decl. ° ′ ″	
12	317 4 23	−15 45 55	Hor. Parallax 59 58
13	317 41 17	−15 38 21	Semi-diameter 16 20

Taking the position of Nanking from the *Connaissance des Temps*, a visible occultation again results:—

	h	m	
Immersion, March 19 at	19	34	Apparent time at Nanking.
Emersion, ,, ,,	20	59	

Consequently the phenomenon occurred within "l'heure *tchin*," as recorded by the Chinese, and the Moon was at the time in the constellation named.

Note by the Astronomer Royal on the Numerical Lunar Theory.

In the *Monthly Notices of the Royal Astronomical Society* for 1874, January, vol. xxxiv., No. 3, I gave as the three equations of motion in the Lunar Theory the three following: (1) The equation of areas described by the radius vector parallel to the plane of the ecliptic; (2) The equation of *vis viva* parallel to the plane of the ecliptic; (3) The equation of motion perpendicular to the plane of the ecliptic.

It now appears preferable to substitute for (2) the equation of radial forces parallel to the plane of the ecliptic, which may be called (2*), and which is easily found to be the following:—

$$\frac{1}{2}\cdot\frac{d^2}{dt^2}\left\{\frac{a'}{a}\cdot\frac{r'}{a'}\cos \mathrm{l}'\right)^2\Big\} - \left(\frac{d}{dt}\left(\frac{a'}{a}\cdot\frac{r'}{a'}\cos \mathrm{l}'\right)\right)^2 - \left(\frac{a'}{a}\cdot\frac{r'}{a'}\cos \mathrm{l}'\right)^2\cdot\left(\frac{dv'}{dt}\right)^2 + \left(\frac{a'}{a}\right)^2\cdot\frac{\epsilon'+\mu'}{a'^3}\cdot\frac{a'}{r'}\cos^2 \mathrm{l}' = +\frac{\mathrm{P}}{a}\cdot\frac{a'}{a}\cdot\frac{r'}{a'}\cdot\cos \mathrm{l}'.$$

and the right-hand side of equation (2*) appears to consist of the following terms:—

$$+\left(e-\frac{eq.\,c.\,f.}{2\,grav.}\right)\cdot\left(\frac{e}{a}\right)^2\cdot\left(\frac{a}{a'}\right)^3\cdot\left(\frac{a'}{r'}\right)^3$$

$$\times\left\{\begin{array}{l}5\,(\sin\omega\,.\cos^2 1'.\sin v'+\cos\omega\,.\sin 1'.\cos 1')^2\\-2\,(\sin\omega\,.\cos^2 1'.\sin v'+\cos\omega\,.\sin 1'.\cos 1')\sin\omega\,.\sin v'\\-\cos^2 1'\end{array}\right\}$$

$$+\frac{\sigma'}{A'^3}\cdot\left(\frac{A'}{R'}\right)^3\cdot\left(\frac{a'}{a}\right)^2\cdot\left(\frac{r'}{a'}\right)^2\times\left\{3\cos^2 1'.\cos^2\overline{|v'-V'|}-\cos^2 1'\right\}$$

$$+\frac{\sigma'}{A'^3}\cdot\frac{\epsilon-\mu}{\epsilon+\mu}\cdot\frac{a}{A'}\cdot\left(\frac{A'}{R'}\right)^4\cdot\left(\frac{a'}{a}\right)^3\cdot\left(\frac{r'}{a'}\right)^3$$

$$\times\left\{-15\cos^3 1'.\cos^3\overline{|v-V|}+\frac{3}{2}\cos 1'.\cos\overline{|v'-V'|}+3\cos^3 1'.\cos\overline{|v'-V'|}\right\}$$

$$+\frac{\sigma'}{A'^3}\cdot\frac{\epsilon^3+\mu^3}{(\epsilon+\mu)^3}\cdot\left(\frac{a}{A'}\right)^2\cdot\left(\frac{A'}{R'}\right)^5\cdot\left(\frac{a'}{a}\right)^4\cdot\left(\frac{r'}{a'}\right)^4$$

$$\times\left\{+\frac{35}{2}\cos^4 1'.\cos^4\overline{|v'-V'|}-\frac{15}{2}\cos^2 1'.\cos^2\overline{|v'-V'}-\frac{15}{2}\cos^4 1'.\cos^2\overline{|v'-V'|}+\frac{3}{2}\cos^2 1'\right\}$$

$$+\frac{\sigma'}{A'^3}\cdot\left(\frac{A'}{R'}\right)^3\cdot\left(\frac{r'}{a'}\right)^2\cdot\cos 1'.\sin 1'\times bt\times 3\,.\cos\overline{|v'-V'|}\,.\sin\overline{|V'-K|}$$

$$+\frac{\sigma'}{A'^3}\cdot\frac{\epsilon-\mu}{\epsilon+\mu}\cdot\frac{a'}{A'}\cdot\left(\frac{A'}{R'}\right)^4\cdot\left(\frac{r'}{a'}\right)^3.\cos^2 1'.\sin 1'\times bt$$

$$\times\left\{-15\,.\cos^2\overline{|v'-V'|}\,.\sin\overline{|V'-K|}+3\sin\overline{|V'-K|}\right\};$$

and there are no terms multiplied by $\frac{dv}{dt}$.

1877, *March* 5.

The Astronomer Royal, in an oral address, adverted to M. Le Verrier's investigations on the possibility of discovery of an intra-mercurial planet, and to his issue of a circular requesting that observations of the Sun's disk for the passage of such planet might be made on March 21, more particularly on March 22, and on March 23. He pointed out that for observation of a transit whose time could not be even approximately predicted, but which certainly would occupy only two or three

hours, it was necessary that observations should be made at widely separated meridians of the earth; and he stated that, at his instance, telegrams had been issued to one station in India, to three in Australia, to two in New Zealand, to Washington, U.S., and also (though it was doubtful whether it could arrive in time) to Santiago di Chili. It was much to be wished that observations could be obtained from San Francisco. He did not doubt that M. Struve would obtain observations in Eastern Siberia, and probably in Japan, and other islands. The Astronomer Royal also remarked that in scrutinizing the Sun's disk with a telescope, it might be possible to pass over some points without observation; and that in this instance photographic pictures, admitting of subsequent long and continuous study, would be preferable to eyeviews in a telescope, in which the power of deciding on the visibility of a spot must terminate with the actual view.

The Astronomer Royal also invited attention to the observations in connection with the determination of the Sun's parallax, which might advantageously be made at the approaching opposition of *Mars*. He cited from the *Monthly Notices* of 1857, his paper "On the means available for correcting our measures of the Sun's distance during the next 25 years," in which, alluding to the method by observing the displacement of *Mars* in right ascension when he is far east and far west of the meridian, and pointing out its facilities, he had concluded that "this method is the best of all." He had endeavoured, in several late Nos. of the *Monthly Notices*, to give facilities for the prosecution of this method, by observations, with an equatoreal, of the transits of *Mars* and stars; but an unexpected opportunity of obtaining observations, probably of a much superior class, has presented itself. Lord Lindsay is willing to lend his heliometer; and Mr. Gill, who has had extensive experience in the use of that instrument, and is perfectly acquainted with its adjustments of all kinds, offers his own time and labour at St. Helena, or Ascension. He proposes to undertake these observations on a guarantee of 500*l.* for expenses; any amount exceeding that sum to be borne by himself. This expense is a trifle compared with that which has been incurred for the transit of *Venus*, which, including the expenses of different nations, may amount to 200,000*l.* The Astronomer Royal suggested that this is an occasion on which the Royal Astronomical Society may well step in with its funds. He mentioned its funds in contradistinction to those of the Government, because the Government has been very liberal, and has felt a great strain put upon it by the expenses in observation of eclipses, and for the transit of *Venus*; and for that very reason the Astronomer Royal is very cautious indeed in suggesting any further application. If the Royal Astronomical Society will not support it from its funds, the Astronomer Royal hopes that the sum may be raised by contributions of individuals, in which he is willing to bear a part [naming a sum].

On the Lunar Perturbations arising from the Planet Jupiter.

By E. Neison.

Shortly after the appearance of Professor Newcomb's paper "On a hitherto unnoticed Inequality in the Longitude of the Moon" (*Monthly Notices*, June 1876, p. 358), my attention was directed to a new term in the lunar longitude arising from the perturbing action of *Jupiter*, and having the value

$$-1''.0 \sin \{(2-2m_1-c)\, nt+f-2f_1+\mathrm{A}\},$$

where $(nt+f)$ is the Moon's mean longitude; $(cnt+f-\mathrm{A})$ the Moon's mean anomaly; and $(m_1 nt+f_1)$ the mean longitude of *Jupiter*. This term agreed well in period with the inequality discovered by Professor Newcomb, and its coefficient was only a little smaller, but was stated to be merely approximate. Being at the time engaged in a similar branch of the lunar theory, I was led to examine this new term and to determine its value with some accuracy.

This term arises from a portion of the disturbing function of the Moon due to the planets, which has not been examined by Laplace, Plana, or Pontécoulant, nor apparently by Hansen, namely, that portion depending on the difference in longitude of the Moon and planet. It is in form analogous to the *evection*, and obtains its sensible value from the approach to equality between the periods of the revolution of the Moon's perigee and of the orbital revolution of the planet *Jupiter*. This fact at once directed my attention to the associated inequality of long period whose argument is—

$$\{(2-2m_1-2c)\, nt-2f_1+2\mathrm{A}\},$$

the value of which is still more dependent on the periods of the revolution of the Moon's perigee and the orbital revolution of the disturbing body.

In the ordinary lunar perturbations arising from the disturbing action of the Sun, the value of the inequality of long period is less than one-twentieth of the associated inequality of short period, because the period of the revolution of the Moon's perigee is nine times as great as the period of the apparent motion of the Sun. In the perturbations due to the disturbing action of the planet *Jupiter*, as the period of the revolution of the Moon's perigee is nearly four-fifths of the period of the revolution of the planet in its orbit, from the much closer approach to equality between the two periods, it is very probable that the inequality of long period would be almost as great as the associated inequality of short period.

The determination of the accurate values of these two terms with algebraical coefficients was found to be a work of unexpected difficulty and much labour. These and similar terms due to *Jupiter* rise by two orders on integration. As the ordinary

lunar perturbations only converge by the same amount for each power of the disturbing force of the Sun, it is necessary to take into consideration the fourth power of the disturbing force of the Sun together with the first power of the disturbing force of *Jupiter*. After satisfying myself that both inequalities possessed a sensible value in comparison with the other inequalities arising from the action of the planets, the task of completely determining the value of these terms was postponed until the completion of my new Tables of the general development of the higher powers of the disturbing forces in the lunar theory.

These Tables have been now finished sufficiently to enable them being used to compute the value of these two inequalities, a work now of comparatively small trouble and difficulty; and the algebraical coefficients obtained, which are lengthy, have been reduced to numbers. The results are—

$$\delta r = -0''\cdot493 \cos\{(2-2m_1-c)nt+f-2f_1+\mathrm{A}\} + 0''\cdot001 \cos\{(2-2m_1-2c)nt-2f_1+2\mathrm{A}\}.$$

$$\delta v = -0''\cdot990 \sin\{(2-2m_1-c)nt+f-2f_1+\mathrm{A}\} + 1''\cdot513 \sin\{(2-2m_1-2c)nt-2f_1+2\mathrm{A}\}.$$

It is of course only in the longitude that these terms could become sensible. The labour in properly computing the value of these terms is shown by the fact that it cannot properly be done unless over twenty similar terms have had their values previously ascertained.

These two new terms derive additional interest from the fact that they are the first terms derived from the second portion of the disturbing force of the planets which have been found to possess sensible coefficients. They, moreover, have considerably larger values than any other inequalities due to *Jupiter*.

Corrections to the Nautical Almanac Places of the Moon for Longitude Observations in connection with the Transit of Venus 1874. By Captain G. L. Tupman.

(*Communicated by the Astronomer Royal.*)

These corrections have been deduced entirely from observations at places which have been connected by galvanic signals with the Royal Observatory, Greenwich, for longitude, and where both limbs of the Moon were equally observed, viz.:—

1° of both elements at Greenwich, with the Transit-Circle and Altazimuth, between 1874, Sept. 15, and 1875, Feb. 23.

2° of both elements at Paris, between 1874, Sept. 14, and 1875, Feb. 22, including those made with the Gambey instruments, published in *Comptes Rendus* 1875, May 24.

3° of Right Ascension at Königsberg between 1874, Sept. 19, and 1875, Feb. 26, published in the *Astronomische Nachrichten*, Nos. 2020 and 2050.

4° of Right Ascension at Strasburg between 1874, Oct. 23, and 1875, Feb. 25, *Astronomische Nachrichten*, No. 2050.

5° of North Polar Distance at the Radcliffe Observatory, Oxford, between 1874, Sept. 18, and 1875, Jan. 21.†

The published errors of the tabular R.A. at Königsberg differ considerably from the errors obtained at the other observatories, and the discordance appears to vary with the Moon's declination, as if the instrumental corrections were imperfect. As, however, the observed R.A.s of the stars observed near the Moon are also given, new errors of the tabular R.A. were determined in the following manner :—

The apparent R.A. of every star at transit over the Meridian of Königsberg, founded on a mean R.A. derived from the Greenwich Catalogue for 1864, or, in default, 1860, was computed and compared with the observed R.A. A correction for the day was thus obtained from each star, and the mean from all the stars was applied to the observed R.A. of the Moon's limb.

The tabular R.A. of the limb was then interpolated with fourth differences from the section "Moon culminating Stars" in the *Nautical Almanac*, adopting for the longitude of Königsberg Observatory $1^h\ 21^m\ 59^s{\cdot}32$, which results from galvanic signals, viz. :—

	m	s	
Brussels east of Greenwich	17	28·90	(*Royal Obs. MSS.*)
Berlin east of Brussels	36	6·42	(*Astr. Nach.* XLV. 225).
Königsberg east of Berlin	28	24·00	(*Astr. Nach.* LVII. 350).

The difference between the interpolated R.A. and the observed R.A. corrected as described above was taken as the error of the tables.

A similar comparison of the observed and tabular R.A. of stars observed near the Moon at Strasburg showed no systematic discordances ; the published corrections were therefore adopted, as were also the Paris corrections.

The mean correction for each day, allowing half weight to the Greenwich Altazimuth and the Königsberg observations, was then plotted off on charts of ruled squares, treating the R.A. and N.P.D. separately, and a free curve drawn through all the points. The adopted position of this curve among the points was submitted to several persons for criticism, and is believed to give the corrections as accurately as the observations admit.

Professor Newcomb's corrections were then plotted off on the same charts, and as the "observation-curve" was weak at the beginning and end of each lunation, it was made parallel to Professor Newcomb's.

The corrections taken from this modified portion of the curve are denoted by an asterisk (*).

† The Washington Observations had not been received at Greenwich when the corrections were determined ; they would have been a valuable addition.

Corrections of the Moon's Tabular Place in the Nautical Almanac.

Day of the Month.	1874 September. R.A. s	N.P.D. "	October. R.A. s	N.P.D. "	November. R.A. s	N.P.D. "	December. R.A. s	N.P.D. "	1875 January. R.A. s	N.P.D. "	February. R.A. s	N.P.D. "
7											−·30	+3·5
8											−·30	+4·2
9											−·31	+4·5
10									−·40*	0·0	−·35	+4·8
11									−·42*	+1·3	−·37	+4·8
12					−·43*	+2·0	−·34*	+2·0	−·44*	+2·5	−·41	+4·5
13	−·37		−·27*	−3·0	−·42*	+2·7	−·35*	+3·0	−·45*	+3·4	−·45	+3·5
14	−·38	−1·0	−·28*	−2·5	−·40*	+3·1	−·36*	+3·8	−·47	+3·8	−·50	+2·3
15	−·40	−0·3	−·30	−2·0	−·37*	+3·4	−·38*	+4·5	−·48	+4·0	−·55	+1·0
16	−·41	0·0	−·35	−1·3	−·35	+3·6	−·40	+5·0	−·50	+3·6	−·57	−0·6
17	−·42	+0·6	−·38	−0·4	−·32	+3·6	−·45	+5·0	−·52	+2·7	−·60	−2·6
18	−·43	+1·3	−·41	+0·7	−·35	+3·6	−·48	+4·5	−·54	+1·4	−·58	−3·6
19	−·44	+1·7	−·44	+2·0	−·37	+3·5	−·51	+3·4	−·56	−0·6	−·55	−4·2
20	−·45	+2·3	−·46	+3·5	−·44	+3·5	−·55	+1·7	−·59	−2·1	−·53	−4·2
21	−·47	+2·8	−·50	+5·0	−·51	+3·3	−·59	0·0	−·62	−3·2	−·48	−3·8
22	−·49	+3·5	−·54	+5·7	−·59	+3·0	−·64	−1·5	−·65	−3·5	−·44	−3·2
23	−·52	+4·1	−·58	+6·0	−·67	+2·6	−·69	−3·0	−·66	−3·6	−·41	−2·0
24	−·56	+5·0	−·65	+5·7	−·75	+2·0	−·74	−4·4	−·66	−3·5	−·39	

Day of the Month.	1874 September. R.A. s	N.P.D. ″	October. R.A. s	N.P.D. ″	November. R.A. s	N.P.D. ″	December. R.A. s	N.P.D. ″	1875 January. R.A. s	N.P.D. ″	February. R.A. s	N.P.D. ″
25	−·61	+5·8	−·75	+5·2	−·85	+1·2	−·75*	−5·3	−·65	−3·3	−·36	
26	−·69	+6·2	−·80	+4·1	−·92	0·0	−·76*	−6·1	−·62	−3·0	−·35	
27	−·77	+5·7	−·80	−2·8	−·91	−1·0	−·73*	−6·7	−·57	−2·6	−·35	
28	−·83	+4·9	−·78	−1·3	−·80	−1·6	−·66*	−7·1	−·53	−2·2		
29	−·83	+4·0	−·76	−0·4	−·66	−2·3	−·60*	−7·1	−·47	−1·7		
30	−·80	+3·0	−·74	−1·6	−·60	−2·8	−·55*	−7·0	−·42	−1·4		
31			−·70	−2·6			−·51*	−6·6	−·40*	−1·0		

	October.		November.		December.		January.		February.	
1	−·75	+1·7	−·66	−3·5	−·55	−3·0	−·48*	−6·0	−·38*	−0·5
2	−·70	+0·6	−·63	−4·1	−·52	−3·1	−·45*	−5·3	−·36*	0·0
3	−·66	−0·7	−·59	−4·4	−·51	−3·0	−·43*	−4·4	−·36*	+0·5
4	−·63	−2·1	−·55	−4·5	−·50	−2·8	−·40*		−·36*	+1·0
5	−·59	−3·2	−·52*	−4·3	−·50*	−2·5				
6	−·56	−3·6	−·48*	−4·0	−·50*	−2·0				
7	−·53*	−4·0								
8	−·51*	−4·0								

The differences between the individual observations and the curve are shown below, the sign + signifying that the observed R.A. and N.P.D. are greater than the adopted.

	Right Ascension.					North Polar Distance.			
1874	Greenwich Transit Circle. s	Greenwich Altazimuth. s	Paris. s	Königsberg. s	Strasburg. s	Greenwich Transit Circle. "	Greenwich Altazimuth. "	Paris. "	Oxford. "
Sept. 14			−·08						
,, 15		+·30	−·11				−3·3		
,, 18		+·40	−·18				+1·5	−2·1	+1·8
,, 19		+·35	$\binom{+·05}{-·19}$	−·08			0·0	+4·2	
,, 20				−·06					
,, 21	+·03	+·19		+·01		−0·2	+0·7	−3·8	+1·5
,, 22	+·02	+·18		−·07		−0·6	−3·0		+2·0
,, 23		−·15		−·10			−1·9		−0·6
,, 24		−·02	−·14				+2·1	−2·0	
,, 25	−·02	+·20	+·06			−1·0	+4·4	−1·3	+0·2
,, 26	+·07	·00	+·05	−·11		−0·9	+0·2	+1·0	
,, 27	+·11	−·34				−0·9	+1·3		
,, 28	+·17	−·30				−1·5	+0·5		
,, 29		−·04		+·04			−0·8		
Oct. 1		−·25					+1·9		
,, 2		−·07					+2·0		
,, 3		+·28					−2·1		

1874	Right Ascension. Greenwich Transit Circle. s	Right Ascension. Greenwich Altazimuth. s	Right Ascension. Paris. s	Right Ascension. Königsberg s	Right Ascension. Strasburg. s	North Polar Distance. Greenwich Transit Circle. ″	North Polar Distance. Greenwich Altazimuth. ″	North Polar Distance. Paris. ″	North Polar Distance. Oxford. ″
Oct. 4		+·01					+1·3		
„ 5		−·53	+·23				−0·9	+0·5	
„ 14			+·04						
„ 15			−·11						
„ 16				+·07					
„ 18	+·11	−·16				+0·8	−1·7		
„ 19	+·06	−·26				+1·5	+1·3		
„ 20	+·02	−·01	−·02			−1·7	−0·4	−3·4	
„ 21	+·12	+·12				+1·5	+3·7		+2·8
„ 22	−·01	−·14				+0·2	−2·1		
„ 23		−·24	·00 −·09		−·06		−2·4	−0·5	
„ 24		+·17	+·04 −·14	−·16	−·11		+0·4	−1·4 +2·6	
„ 25		−·26		−·07	−·24		−2·3	+4·2	
„ 26			+·01					+2·5	
„ 27	+·23	−·20	−·20		−·03	−1·0	−1·1	+2·1	
„ 28	+·05	−·06				−1·5	−0·9		
„ 29				·00					
„ 30		−·24		+·19			−2·4		

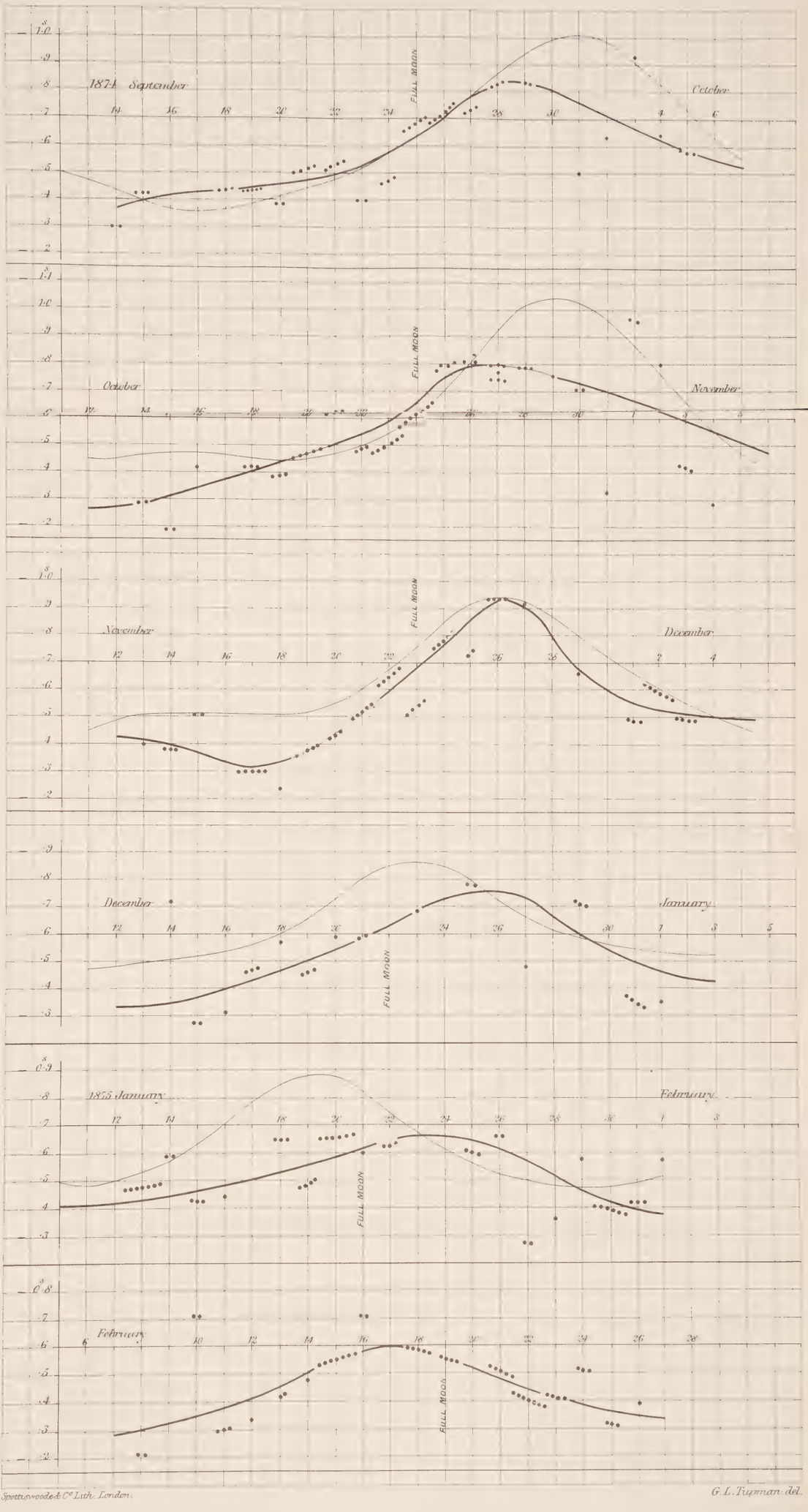
Oct. 31		−·37					−1·6		
Nov. 1			+·29					+1·1	
,, 2		+·17					−0·1		
,, 3		−·22	−·15				+0·7	−1·3	
,, 4		−·28					+0·9		
,, 13		−·02					−3·1		
,, 14			−·14	+·21				+1·2	+2·1
,, 15					+·14				
,, 17	+·11	−·21	−·05			+1·0	−4·0		
,, 18		−·11					−6·0		
,, 19	+·12	+·14	−·18			+1·2	+3·1	−0·9	−3·4
,, 20	+·07	−·18				−2 4	−1·1		
,, 21			(+·22 −·22)		+·13			(+2·1 −1·1)	
,, 22	−·16	+·14			−·08	+0·8	+2·0		
,, 23		−·16	(−·02 −·17)				+1·9	(−3·1 +1·8)	−2·6
,, 24		+·23	+·04		−·07		+5·1		
,, 25			−·11					+0·3	
,, 26	−·07				+·10	−5·9			
,, 27		+·01					+1·2		
,, 29		−·01					+0·3		
Dec. 1	−·01	−·18				+1·4	−0 5		

	Right Ascension.					North Polar Distance.			
	Greenwich Transit Circle.	Greenwich Altazimuth.	Paris.	Königsberg.	Strasburg.	Greenwich Transit Circle.	Greenwich Altazimuth.	Paris.	Oxford.
	s	s	s	s	s	″	″	″	″
1874 Dec. 2	+·19	·00	−·02		+·06	−1·6	+2·5		
,, 3		−·20	+·07		−·05		−0·2	−0·6	
,, 14		+·36					−2·1		+0·7
,, 15			−·11						
,, 16		−·09					−0·4		+0·6
,, 17	+·07	−·08				−1·3	+2·6		
,, 18		+·09					+0·4		
,, 19	+·01	−·18				−0·4	−0·5		+2·0
,, 20		+·04					−2·5		
,, 21	·00					+1·3	−2·8		+1·1
,, 23		·00					+0·8		
,, 25	+·03					−0·5			
,, 27		−·26					+1·7		
,, 29	+·23	−·13				0·0	−1·5		
,, 31		−·21	−·16	−·11			−2·5	+1·3	
1875 Jan. 1		+·13							
,, 13	−·05	+·26	−·17	+·17		−1·1	−0·4	+0·3	−3·9
,, 14				+·12					
,, 15	+·06	−·28				+0·1	+0·2		−0·6

Jan. 16		−·05					+ 1·5		− 1·5
„ 17		−·34					0·0		
„ 18	+ ·05	+ ·22				− 1·6	+ 1·2		− 6·4
„ 19		− ·14	(+ ·04 / − ·09)				+ 0·7	(+ 1·3 / + 2·5)	
„ 20	+ ·28	+ ·06	(+ ·04 / − ·11)			− 1·6	+ 1·6	(− 3·8 / 0·0)	− 4·3
„ 21		−·02				− 1·6	+ 1·4		− 3·2
„ 22		+ ·14			− ·09		+ 3·3		
„ 24		+ ·33					− 0·5		
„ 25		−·30			+ ·07		+ 0·6		
„ 26					+ ·04				
„ 27					− ·30				
„ 28				− ·17					
„ 29		+ ·10					0·0		
„ 30		+ ·03	− ·10	− ·25	+ ·13	− 0·5	+ 1·5		
„ 31		−·20	+ ·14				− 0·6		
Feb. 1		+ ·19					− 6·5		
„ 8			− ·09						
„ 10					+ ·37				
„ 11			− ·02	− ·18					
„ 12		− ·07							
„ 13				− ·03					

	Right Ascension.					North Polar Distance.			
	Greenwich Transit Circle.	Greenwich Altazimuth.	Paris.	Königsberg.	Strasburg.	Greenwich Transit Circle.	Greenwich Altazimuth.	Paris.	Oxford.
1875	s	s	s	s	s	″	″	″	
,, 14		−·02					−3·1		
,, 15	+·15	−·24	(+·03 / −·10)	+·08		−0·3	+4·0	(−1·8 / +1·2)	
,, 16		+·30		−·02			−2·4		
,, 17		−·12					+1·2		
,, 18	+·08	+·13	(+·21 / −·18)			−1·6	+1·6	(−0·3 / +0·2)	
,, 19			(−·01 / −·05)	+·11				(−0·8 / −0·1)	
,, 21	+·12	−·16			+·04	+1·0	−0·5		
,, 22	+·09	−·16	−·15		·00	−1·1	−2·4	+0·1	
,, 23		−·25		−·20	+·23		+3·0		
,, 24				−·12	+·24				
,, 25				−·15	+·07				
,, 26				+·04					

CORRECTIONS TO THE MOON'S TABULAR RIGHT ASCENSION.



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EXPLANATION OF THE ENGRAVED CURVES.

The dots represent the actual observations as weighted in the following manner:-

The observations with the Greenwich Transit Circle, at Paris, and at Strasburg, have the weight 2; those at Königsberg and with the Greenwich Altazimuth the weight 1.

The number of dots shows the weight for the day; the centre dot of each group should be on the ordinate representing the day.

The strong black line is the adopted correction, the fine line shows the correction deduced by Professor Newcomb.

On the Visibility of the Unilluminated Portion of the Disk of Venus. By Señor A. T. Arcimis.

I had only one favourable opportunity of observing this phenomenon in the afternoon of June 19, 1876; on the previous and following days the sky was overcast towards the West, and the prevailing high wind caused considerable vibration of the telescope.

At $7^h\ 15^m$ the Sun being slightly obscured by clouds, I could see the planet with the naked eye; the finder enabled me to distinguish perfectly the shape and direction of the crescent. It presented, when viewed through the telescope provided with an eye-piece of 92 diameters, a most beautiful aspect, although it was very difficult to focus, and the image almost invariably was irisated. The concave portion appeared as a broken or undulating line, somewhat similar to the Moon's, and no illuminated or bright cusps were visible on the dark portion.

The two horns were easily distinguishable. They were unequal, the northern being the sharpest. Close to the southern extremity an excavation or indentation of an almost black colour was perceptible. I could not discard the idea that the body under observation was a partially illuminated sphere; and this appearance became more marked when I employed a very faint red modifier. The phenomenon of irradiation seemed to me more striking than the Moon's; that is to say, the bright portion appeared very considerably larger than the dark one.

I had considerable difficulty in perceiving the secondary light, but I succeeded in doing so by employing the well-known device of not sighting it directly, so that the light might impinge on a more impressionable point of the retina. Its colour, to my sight, was greyish, or somewhat violaceous, and extremely faint. It will be well to bear in mind that this colour might be due to a deficient achromatic condition of the telescope.

At $7^h\ 30^m$ both horns appeared to be equal, and from that moment the sky was overcast, and I have since had no further opportunity of observing the planet.

Cadiz,
1876, *December* 30.

Phenomena of Jupiter's Satellites observed at Cadiz. By Señor A. T. Arcimis.

The observations were made with a 4-inch Secretan Refractor and a 92-power eye-piece. The hour of the clock was determined by equal altitudes of the Sun or stars, taken with the sextant and the artificial horizon. The ephemerides are from the *Nautical Almanac* of San Fernando Observatory; the geographical co-ordinates of my Observatory are:—

Lat. N. $= 36^\circ\ 31'\ 52''.8$

Long. W. Greenwich $= 0^h\ 25^m\ 11^s.7$

Date of Observation.	Satellite.	Phase.	Mean Solar Time of Observation.	Mean Solar Time from N.A.	Diff.	Remarks.
1876.			h m s	h m s	m s	
April 24	I	D	8 9 4	8 9 2	−0 2	Moon absent; clear sky.
„ 25	II	D	14 43 39	14 43 40	+0 1	do. do.
„ 26	III	D	11 16 55	11 16 52	−0 3	Good observation.
„ 26	III	R	12 58 43	12 58 35	−0 8	do.
May 1	I	D	10 2 53	10 2 51	−0 2	do.
„ 3	III	D	15 15 1	15 14 53	−0 8	Bad definition.
„ 8	I	D	11 56 12	11 56 44	−0 32	Clear sky; Moon bright near planet; satellite very tremulous; I lost it sometimes.
„ 20	II	R	14 16 30	14 16 26	−0 4	Good observation.
„ 24	I	R	12 21 7	12 21 1	−0 6	do.
June 1	III	R	8 51 3	8 51 12	+0 9	Bright Moon.
„ 7	II	R	8 47 40	8 46 46	−0 54	Bright Moon; clear sky.
„ 8	III	D	11 7 28	11 5 40	−1 48	Good observation.
„ 8	III	R	12 48 17	12 50 3	+1 46	do.
„ 9	I	R	10 38 19	10 38 5	−0 14	Good obs.; clear sky.
„ 16	I	R	12 32 21	12 32 28	+0 7	do.
„ 25	I	R	8 57 29	8 55 33	−1 56	Bad obs.; strong wind.
July 2	I	R	10 50 23	10 50 6	−0 17	Good obs.; planet near Moon.
„ 14	III	D	not obs.	6 59 33		
„ 14	III	R	8 49 33	8 46 57	−2 36	Strong wind; telescope vibrates.
„ 16	I	R	14 37 48	14 39 23	+1 35	do do.
„ 16	II	R	11 11 52	11 9 35	−2 17	do. do.
„ 18	I	R	9 14 24	9 8 5	−6 19	Clear sky.
„ 25	I	R	11 1 6	11 2 48	+1 42	Cloudy; planet near horizon.
Aug. 10	II	R	8 23 0	8 21 48	−1 12	Very strong wind; bad observation.
„ 10	I	R	9 21 17	9 20 58	−0 19	do. do.
„ 26	I	R	7 39 16	7 39 11	−0 5	Good observ.; Moon present.
„ 26	III	D	6 53 59	6 53 21	−0 38	Good obs.; twilight.
„ 26	III	R	8 45 42	8 45 9	−0 33	Good observ.; Moon present.

Observations of Occultations of Stars by the Moon, made at Cadiz in the year 1876. By Señor A. T. Arcimis.

Date of Observation. 1876	Stars.	Mag.	Phase.	Mean Solar Time of Observation. h m s	Mean Solar Time from *N.A.* h m s	Diff. m s	Remarks.
May 27	γ Cancri	4	D	8 45 47	8 45 38	−0 9	Dark limb.
,, 27	γ Cancri	4	R	9 42 5	9 41 38	−0 27	
Aug. 28	C.A.B. 6107	4	D	9 26 7	9 26 38	+0 31	
,, 28	C.A.B. 6107	4	R	10 20 15	10 19 38	+0 23	Clear sky; dark limb; Moon in □.
Sept. 29	ι Aquarii	4	D	13 54 3	13 53 38	−0 25	Good observation.
,, 29	ι Aquarii	4	R	14 22 17	14 26 38	+4 21	Moon near horizon.
Oct. 6	17 Tauri	4	D	7 57 2	7 56 38	−0 24	Bright limb.
,, 6	17 Tauri	4	R	8 44 10	8 45 38	+1 28	Dark limb.
,, 6	20 Tauri	5	D	8 39 41	8 35 38	−4 3	Bright limb; vapours.
,, 6	20 Tauri	5	R	9 7 23	9 5 38	−1 45	Dark limb; good observation.
,, 6	23 Tauri	5	D	8 25 4	8 27 38	+2 34	Bright limb; bad observation.
,, 6	23 Tauri	5	R	9 2 51	9 1 38	−1 13	Dark limb; bad observation.
,, 6	η Tauri	3	D	8 53 32	8 53 38	+0 6	Bright limb; bad observation.
,, 6	η Tauri	3	R	9 38 9	9 37 38	−0 31	Dark limb; good observation.
,, 26	δ Capricorni	3	D	12 0 33	12 1 38	+1 5	Dark limb; good obs.; Moon near horizon.

The Occultation of Regulus, February 26, 1877.

By F. C. Penrose, Esq.

The occultation of Regulus, Feb. 26, 1877, was observed here.

	h	m	s	
Disappearance	12	46	14·8	G.M.T.
Reappearance	13	51	5·6	

Two observers watched for the disappearance; one with a $2\frac{1}{4}$-inch O. G., the other with a $5\frac{1}{4}$-inch O. G. The clouds were sufficiently dense to obscure the star in the $2\frac{1}{4}$-inch before the occultation, but it was distinctly seen in the $5\frac{1}{4}$-inch on the dark limb. At the reappearance the sky was perfectly clear, and the star was seen as a bright indent upon the limb of the Moon.

Colebyfield, Wimbledon.

Latitude 51° 24′ 58″.

Longitude $0^h\ 0^m\ 55^s{\cdot}15$.

Total Eclipse of the Moon, February 27, 1877.

By F. C. Penrose, Esq.

I offer a drawing made of the Total Eclipse of the Moon on the 27th ulto., at the phase of reappearance of the sunlight. It could not have failed to be remarked during totality, how very much brighter the northern and southern parts of the Moon's limb were than the eastern and western parts; presumably arising from some interference with the light refracted through the Earth's atmosphere, from a greater prevalence of clouds in the equatoreal regions than at the two poles. Some amount of *specular* reflection, at very flat angles, from the northern and southern ice, may have also aided in producing this effect.

With respect to the general phenomenon of the total lunar eclipse, I would observe that, although no value appears in works on Navigation to be placed on this phenomenon for the determination of longitude at sea, although I admit that this would be perfectly reasonable in the case of a partial eclipse; yet, when the eclipse is total, it seems quite capable of giving a useful result. The two phases of the beginning and end of totality should be observed when practicable, and the mean taken. By combining these observations in the total lunar eclipse of July 12, 1870, I obtained a G.M.T. differing from the prediction in the *Nautical Almanac* by 39^s, and from the eclipse of last February by 29^s, both being later than the prediction. There would be less difference than might be supposed in the use of smaller telescopes; for I found that simultaneous unaided eye observations of both phases made on the last occasion did not differ by many seconds. This could, after a little study, be allowed for; and probably, as far as the observation is concerned, the error need not exceed 15 or 20 seconds. The errors of the tables

would of course be shared by this with any other lunar method. The times I noted were,

	h	m	s	
Totality commenced	6	28	13	G.M.T.
Totality ended	8	3	15	

The colour seen upon the Moon on this occasion, and especially the remarkable bluish hue which spread over the lunar regions preceding the reappearance of the sunlight, could not fail strongly to impress upon the mind the wish to realise in imagination the effect that would be seen from the Moon of the prismatic colours surrounding our planet, with the Sun's corona in the background and with *Venus* not far distant.

Total Eclipse of the Moon, February 27, 1877.

By the Rev. S. J. Perry.

The night was cloudless, but there was a sharp frost, which made the air unsteady. The Moon rose partially eclipsed. Using a power of about 100 with an 8-inch achromatic, the following contacts were observed :—

	h	m	s	
Commencement of Totality	6	27	24·5	G.M.T.
End of Totality	8	4	8·5	
Last contact with Shadow	9	0	6·0	

The time was taken with a Frodsham chronometer, compared during the eclipse with the standard sidereal clock; and clock stars were observed.

The darkness of that portion of the penumbra which was in close proximity to the umbra was so great, that the last contact was difficult to observe.

Several reappearances of occulted stars were seen during totality, and might have been well observed.

The thin circle of light on the Moon's limb was in such striking contrast with the cloudy dull brick-red shading of the centre, that to many persons it seemed as if the Moon was not completely immersed in the Earth's shadow.

Stonyhurst Observatory,
1877, *February* 28.

On Kepler's Problem. By M. A. de Gasparis.

(*Translation.*)

The numerical solution of the equation $M = E - e'' \sin E$ may be useful, especially for the new planets, for comets of elliptic orbit, and for the satellites of double stars, the number of which

is always increasing. I here reproduce a solution, very much simplified, which, proceeding by direct operations, gives to within a few seconds the value of E.

I assume $E = M + n.10°$, and we then have by substitution

$$n\frac{(10°)''}{R''} = e \sin (M + n.10°), \quad \ldots\ldots\ldots \quad (1)$$

or, if we please,

$$10n\frac{(1°)''}{R''} = e \sin (M + 10n.1°) \quad \ldots\ldots\ldots \quad (2)$$

The unknown quantity is here n, and we know that its integer part is less than 6. For a first step I employ logarithms to three places of decimals, for the second step to five places. Moreover $\log \frac{(10°)''}{R''} = 9{\cdot}2418774$, and therefore $\log \frac{(1°)''}{R''} = 8{\cdot}2418774$ This being so,

1° I write in a line the logarithms of 1, 2, 3, 4, 5, 6.

2° Below these the logarithm of $\frac{(10°)''}{R''}$, and I take the sums.

This operation is common to all the examples which present themselves.

3° On another line, respectively below the preceding numbers, I write (six times) $\log e$.

4° Below the $\log e$'s, I write the log sin $(M+1.10°)$, log sin $(M+2.10°)$. . . log sin $(M+6.10°)$; and I take the sums.

We have at sight the limits of the integer part of n, and the tenths are obtained by a simple proportion.

Suppose, for example, $M = 27°\ 15'\ 7''{\cdot}42$, $\log e = 9{\cdot}9231544$, we proceed as follows:—

	1	2	3	4	5	6
$\log n$	0·000	0·301	0·477	0·602	0·699	0·778
$\log \frac{(10°)''}{R''}$	9·242	9·242	9·242	9·242	9·242	9·242
	9·242	9·543	9·719	9·844	9·941	0·020
$\log e$	9·923	9·923	9·923	9·923	9·923	9·923
log sin $(M+n.10°)$	9·782	9·866	9·925	9·965	9·989	0·000
	9·705	9·789	9·848	9·888	9·912	9·923

Therefore n is between 4 and 5. The lower sum of the vertical line 4 is greater than the upper sum by 44; on the line 5 it is less by 29; therefore $n = 4 + \frac{44}{44+29} = 4{\cdot}6$. We have consequently

$E = 27° 15' + 46° = 73° 15'$. Since $10n = 46$, I employ in the second case the equation (2) as follows:—

log $10n$	1·65321	1·66276	1·67210
log $\frac{(1°)''}{R''}$	8·24188	8·24188	8·24188
	9·89509	9·90464	9·91398
log e	9·92315	9·92315	9·92315
log sin $(M + 10n.1°)$	9·97882	9·98117	9·98338
	9·90197	9·90432	9·90653

Here $10n = 45 + \frac{688}{688+32} = 45·956$; but $45°·956 = 45° 57' 22''$. Therefore $E = 27° 15' 7''·4 + 45° 57' 21''·6$; or, $E = 73° 12' 29''$, a value exact to within a few seconds.

The definitive correction ΔE, may be obtained from

$$\Delta E = \frac{M + e \sin E - E}{1 - e \cos E}.$$

Naples, Feb. 7, 1877.

Note on a Transformation of Lagrange's Equations of Motion in Generalised Coordinates, which is convenient in Physical Astronomy. By Robert S. Ball, LL.D., F.R.S., Royal Astronomer of Ireland.

I can hardly suppose that the following Note contains anything which has not already been published; but as I have not met with this transformation in my reading, nor conversed with anyone who is acquainted with it, I am led to believe that, even if it be not new, it is certainly not so well known as its convenience deserves.

Lagrange's Equations of Motion in Generalised Coordinates are not, in their usual form, particularly well adapted for Physical Astronomy. In a problem in Physical Astronomy, what we want to learn is the *relative* coordinates of a system, while its *absolute* situation in space does not concern us. Lagrange's equations, however, involve *both* the relative and the absolute coordinates, and we are obliged to remove the latter by elimination. It is the object of this Note to point out a very simple transformation of Lagrange's equations which obviates the inconvenience of having to perform the elimination. This advantage is gained without detracting from the simplicity and directness which render Lagrange's equations such exquisite instruments of analysis.

If q be one of the generalised coordinates, and T and V be the kinetic and potential energies of the system, then Lagrange's equations have the form

$$\frac{d}{dt}\left(\frac{d\mathrm{T}}{d\dot{q}}\right)-\frac{d\mathrm{T}}{dq}=-\frac{d\mathrm{V}}{dq};$$

V depends solely upon the relative coordinates of the system. It is therefore only the function T which requires to be purged from the three objectionable quantities. This is effected by simply replacing T by a different function, which we shall denote by T′.

We proceed to show that this substitution is allowable, and that it answers the end for which it is intended.

Let p_1, p_2, p_3 be the rectangular coordinates of the centre of inertia P of one of the bodies with reference to axes fixed in space. Let the relative generalised coordinates of the system be $q_1, \ldots q_{n-3}$. When the last-mentioned coordinates and the corresponding generalised velocities are known, then everything is known about the system except its actual situation in space. What we want to obtain is a system of $n-3$ equations in $q_1, \ldots q_{n-3}$, from which p_1, p_2, p_3 are absent.

Let x_1, y_1, z_1 be the rectangular coordinates of the centre of inertia of one of the masses m_1 with respect to three rectangular axes through P parallel to the fixed axes, then we have

$$\mathrm{T}=\frac{1}{2}\Sigma m_1(\dot{p}_1+\dot{x}_1)^2+\frac{1}{2}\Sigma m_1(\dot{p}_2+\dot{y}_1)^2+\frac{1}{2}\Sigma m_1(\dot{p}_3+\dot{z}_1)^2+\mathrm{R}_o,$$

when R_o is that function of $q_1, \ldots q_{n-3}$ and $\dot{q}_1, \ldots \dot{q}_{n-3}$ which expresses the kinetic energy of the rotations of the bodies of the system about their centres of inertia.

Then we have

$$\mathrm{T}=\frac{1}{2}(\dot{p}_1^2+\dot{p}_2^2+\dot{p}_3^2)\Sigma m_1+\dot{p}_1\Sigma m_1\dot{x}_1+\dot{p}_2\Sigma m_1\dot{y}_1+\dot{p}_3\Sigma m_1\dot{z}_1$$

$$+\frac{1}{2}(\dot{x}_1^2+\dot{y}_1^2+\dot{z}_1^2)\Sigma m_1+\mathrm{R}_o.$$

We first form the generalised equations of motion in Lagrange's form corresponding to the coordinate p_1:

$$\frac{d\mathrm{T}}{d\dot{p}_1}=\dot{p}_1\Sigma m_1+\Sigma m_1\dot{x}_1,$$

$$\frac{d\mathrm{T}}{dp_1}=0,$$

$$\frac{d\mathrm{V}}{dp_1}=0;$$

whence the equation of motion is

$$\frac{d}{dt}(\dot{p}_1\Sigma m_1+\Sigma m_1\dot{x}_1)=0,$$

and two similar equations, corresponding to p_2 and p_3. If then u, v, w be constants with respect to the time,

$$\dot{p}_1 \Sigma m_1 + \Sigma m_1 \dot{x}_1 = u \Sigma m_1,$$

$$\dot{p}_2 \Sigma m_1 + \Sigma m_1 \dot{y}_1 = v \Sigma m_1,$$

$$\dot{p}_3 \Sigma m_1 + \Sigma m_1 \dot{z}_1 = w \Sigma m_1.$$

We now proceed to form the equations in $q_1, \ldots q_{n-3}$. We shall use the letter q as typical of the series.

If we make

$$T' = \frac{1}{2} (\dot{x}_1^2 + \dot{y}_1^2 + \dot{z}_1^2) \Sigma m_1$$

$$- \frac{1}{2 \Sigma m_1} \{(\Sigma m_1 \dot{x}_1)^2 + (\Sigma m_1 \dot{y}_1)^2 + (\Sigma m_1 \dot{z}_1)^2\} + R_o,$$

then it is easily verified that

$$T = \frac{1}{2} (u^2 + v^2 + w^2) \Sigma m_1 + T'.$$

Making this substitution in the equation of motion corresponding to q, viz.

$$\frac{d}{dt}\left(\frac{dT}{d\dot{q}}\right) - \frac{dT}{dq} = -\frac{dV}{dq},$$

the equation becomes

$$\left\{\frac{d}{dt}\left(\frac{du}{d\dot{q}}\right) - \frac{du}{dq}\right\} u \Sigma m' + \left\{\frac{d}{dt}\left(\frac{dv}{d\dot{q}}\right) - \frac{dv}{dq}\right\} v \Sigma m' + \left\{\frac{d}{dt}\left(\frac{dw}{d\dot{q}}\right) - \frac{dw}{dq}\right\} w \Sigma m'$$

$$+ \frac{d}{dt}\left(\frac{dT'}{d\dot{q}}\right) - \frac{dT'}{dq} = -\frac{dV}{dq}.$$

But this equation may be greatly simplified, for

$$u = \frac{d}{dt}\left(p_1 + \frac{\Sigma m_1 x_1}{\Sigma m_1}\right);$$

or, if we assume

$$U = \frac{\Sigma m_1 x_1}{\Sigma m_1},$$

we have

$$u = \dot{p}_1 + \frac{dU}{dt};$$

but U is a function of $q_1, \ldots q_{n-3}$, and does not contain t or $\dot{q}_1, \ldots \dot{q}_{n-3}$, or p_1, p_2, p_3, whence we may write

$$u = \dot{p}_1 + \Sigma \frac{dU}{dq} \dot{q}.$$

It follows that

$$\frac{du}{d\dot{q}} = \frac{d\mathrm{U}}{dq},$$

$$\frac{du}{dq} = \frac{d^2\mathrm{U}}{dt\,dq};$$

and finally

$$\frac{d}{dt}\left(\frac{du}{d\dot{q}}\right) - \frac{du}{dq} = 0.$$

Similarly it may be shown that

$$\frac{d}{dt}\left(\frac{dv}{d\dot{q}}\right) - \frac{dv}{dq} = 0,$$

$$\frac{d}{dt}\left(\frac{dw}{d\dot{q}}\right) - \frac{dw}{dq} = 0;$$

and therefore the equation of motion reduces to

$$\frac{d}{dt}\left(\frac{d\mathrm{T}'}{d\dot{q}}\right) - \frac{d\mathrm{T}'}{dq} = -\frac{d\mathrm{V}}{dq}.$$

By slightly altering the method of expressing the function T, the final result may be stated in the following theorem:—

Let V denote the potential energy of a system of attracting bodies.

Let T′ denote the sum of the kinetic energies of the rotations of each body of the system about its centre of inertia added to

$$\Sigma \frac{m_1 m_2}{2\Sigma m_1}[1, 2],$$

where the symbol [1, 2] denotes the square of the relative velocity of the centres of inertia of the masses m_1 and m_2, and the summation includes every pair of the bodies of the system.

Let $q_1, \ldots q_{n-3}$ be $n-3$ generalised coordinates adequate to specify everything about the position and aspect of the system at a given epoch, except its actual situation in space, then the $n-3$ equations which will determine the motion are

$$\frac{d}{dt}\left(\frac{d\mathrm{T}'}{d\dot{q}_1}\right) - \frac{d\mathrm{T}'}{dq_1} = -\frac{d\mathrm{V}}{dq_1},$$

&c.

$$\frac{d}{dt}\left(\frac{d\mathrm{T}'}{d\dot{q}_{n-3}}\right) - \frac{d\mathrm{T}'}{dq_{n-3}} = -\frac{d\mathrm{V}}{dq_{n-3}}.$$

In these equations the three objectionable coordinates do not appear, and thus the required elimination has been effected.

Addition by Prof. Cayley.

The formulæ may be established in a somewhat different way, as follows:—

Consider the masses M_1, M_2, . . .

Let X_1, Y_1, Z_1 be the coordinates (in reference to a fixed origin and axes) of the C. G. of M_1;

x_1, y_1, z_1 the coordinates (in reference to a parallel set of axes through the C. G. of M_1) of an element m_1 of the mass M_1, and similarly for the masses M_2, . . .; the coordinates (X_1, Y_1, Z_1), (X_2, Y_2, Z_2), . . . all belonging to the same origin and axes;

And let $\dot{X}_1$ &c. denote the derived functions $\frac{dX_1}{dt}$ &c.

We have

$$T = \quad S\tfrac{1}{2} m_1 [(\dot{X}_1+\dot{x}_1)^2+(\dot{Y}_1+\dot{y}_1)^2+(\dot{Z}_1+\dot{z}_1)^2]$$

$$+S\tfrac{1}{2} m_2 [(\dot{X}_2+\dot{x}_2)^2+(\dot{Y}_2+\dot{y}_2)^2+(\dot{Z}_2+\dot{z}_2)^2];$$

$$\vdots$$

or since $S m_1 x_1 = 0$ &c., and therefore also $S m_1 \dot{x}_1 = 0$ &c., this is

$$T = \tfrac{1}{2} M_1 (\dot{X}_1^2+\dot{Y}_1^2+\dot{Z}_1^2) \qquad +S\tfrac{1}{2} m_1 (\dot{x}_1^2+\dot{y}_1^2+\dot{z}_1^2)$$

$$+\tfrac{1}{2} M_2 (\dot{X}_2^2+\dot{Y}_2^2+\dot{Z}_2^2) \qquad +S\tfrac{1}{2} m_2 (\dot{x}_2^2+\dot{y}_2^2+\dot{z}_2^2)$$

$$\vdots \qquad\qquad\qquad\qquad \vdots$$

Write u, v, w for the coordinates of the C. G. of the whole system, then

$$M_1 X_1+M_2 X_2+ \cdot\cdot = (M_1+M_2 \cdot\cdot)\, u,$$

$$M_1 Y_1+M_2 Y_2+ \cdot\cdot = (M_1+M_2 \cdot\cdot)\, v,$$

$$M_1 Z_1+M_2 Z_2+ \cdot\cdot = (M_1+M_2 \cdot\cdot)\, w;$$

and thence

$$M_1 \dot{X}_1+M_2 \dot{X}_2+ \cdot\cdot = (M_1+M_2 \cdot\cdot)\, \dot{u},$$

$$M_1 \dot{Y}_1+M_2 \dot{Y}_2+ \cdot\cdot = (M_1+M_2 \cdot\cdot)\, \dot{v},$$

$$M_1 \dot{Z}_1+M_2 \dot{Z}_2+ \cdot\cdot = (M_1+M_2 \cdot\cdot)\, \dot{w};$$

and thence

$$T-\tfrac{1}{2}(M_1+M_2+ \cdot\cdot)(\dot{u}^2+\dot{v}^2+\dot{w}^2)$$

$$= \frac{1}{M_1+M_2 \cdot\cdot} \{M_1 M_2 [(\dot{X}_1-\dot{X}_2)^2+(\dot{Y}_1-\dot{Y}_2)^2+(\dot{Z}_1-\dot{Z}_2)^2]\}$$

$$\vdots$$

$$+S\tfrac{1}{2} m_1 (\dot{x}_1^2+\dot{y}_1^2+\dot{z}_1^2)$$

$$+S\tfrac{1}{2} m_2 (\dot{x}_2^2+\dot{y}_2^2+\dot{z}_2^2);$$

$$\vdots$$

or, representing the function on the right-hand side by T′, this is

$$T = \frac{1}{2}(M_1 + M_2 + \cdots)(\dot{u}_2 + \dot{v}_2 + \dot{w}_2) + T' \ldots, = T_0 + T'.$$

Suppose the positions are determined by means of the $6n$ coordinates $((q))$; the equations of motion are each of them of the form

$$\frac{d}{dt}\cdot\frac{dT_0}{d\dot{q}} - \frac{dT_0}{dq} + \frac{d}{dt}\cdot\frac{dT'}{d\dot{q}} - \frac{dT'}{dq} = -\frac{dV}{dq};$$

but these admit of further reduction; the part in T_0 depends upon three terms, such as

$$\frac{d}{dt}\left(\dot{u}\frac{d\dot{u}}{d\dot{q}}\right) - \dot{u}\frac{d\dot{u}}{dq}, = \frac{d\dot{u}}{dt}\frac{d\dot{u}}{d\dot{q}} + \dot{u}\left(\frac{d}{dt}\frac{d\dot{u}}{d\dot{q}} - \frac{d\dot{u}}{dq}\right).$$

But we have u a function of $((q))$, and thence

$$\frac{d\dot{u}}{d\dot{q}} = \frac{du}{dq}, \text{ or } \frac{d}{dt}\frac{d\dot{u}}{d\dot{q}} - \frac{d\dot{u}}{dq}, = \frac{d}{dt}\frac{du}{dq} - \frac{d\dot{u}}{dq}, = 0,$$

or the term is simply

$$= \frac{d\dot{u}}{dt}\frac{d\dot{u}}{dq}.$$

The equation thus becomes

$$(M_1 + M_2 \ldots)\left(\frac{d\dot{u}}{dt}\frac{d\dot{u}}{d\dot{q}} + \frac{d\dot{v}}{dt}\frac{d\dot{v}}{d\dot{q}} + \frac{d\dot{w}}{dt}\frac{d\dot{w}}{d\dot{q}}\right) + \frac{d}{dt}\frac{dT'}{d\dot{q}} - \frac{dT'}{dq} = -\frac{dV}{dq}.$$

Suppose now that T′, V are functions of $6n-3$ out of the $6n$ coordinates $((q))$, and of the differential coefficients $\dot{q}$ of the same $6n-3$ coordinates, but are independent of the remaining three coordinates and of their differential coefficients; then, first, if q denotes any one of the three coordinates, the equation becomes

$$\frac{d\dot{u}}{dt}\frac{d\dot{u}}{d\dot{q}} + \frac{d\dot{v}}{dt}\frac{d\dot{v}}{d\dot{q}} + \frac{d\dot{w}}{dt}\frac{d\dot{w}}{d\dot{q}} = 0;$$

or, better,

$$\frac{d\dot{u}}{dt}\frac{du}{dq} + \frac{d\dot{v}}{dt}\frac{dv}{dq} + \frac{d\dot{w}}{dt}\frac{dw}{dq} = 0;$$

and the three equations of this form give

$$\frac{d\dot{u}}{dt} = 0, \quad \frac{d\dot{v}}{dt} = 0, \quad \frac{d\dot{w}}{dt} = 0,$$

viz., these are the equations for the conservation of the motion of the centre of gravity.

And this being so, then, if q now denotes any one of the $6n-3$ coordinates, each of the remaining equations assumes the form

$$\frac{d}{dt}\cdot\frac{dT'}{d\dot{q}}-\frac{dT'}{dq}=-\frac{dV}{dq},$$

viz., we have thus $6n-3$ equations for the relative motion of the bodies of the system.

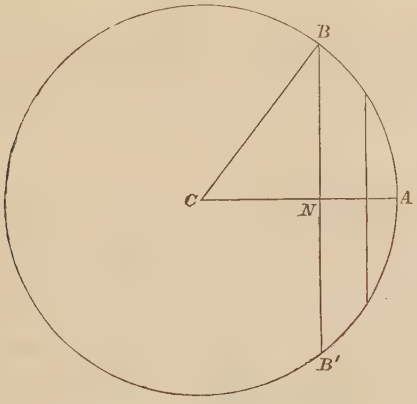
On Photographing Solar Transits by the use of the "Starlit Transit Eye-piece" formerly described; and other methods. By Dr. Royston-Pigott, M.A., F.R.S.

It may perhaps be recollected that the eye-piece contains a ruled micrometer, displaying equidistant parallel lines drawn on a silver film with great accuracy. At present the Sun transits in a five-feet telescope from line to line in five seconds—a second per foot of focal length.

If the image of the Sun in transits be projected on a screen of paper, a peculiar phenomenon is observed worth noting. So soon as the solar light illuminates the field, the transparent lines remain *black* until the sun flashes across the lines.

On the instant that the limb makes contact, a bright line is seen rapidly lengthening as a brilliant tangent, becoming a chord to the disk.

During the passage of the Sun across the bars, at each minute change of position the phases of the bars pass through a continual series of varying patterns.



If now an instantaneous photograph be taken at a known instant of time, the measured length of any one of these brilliant

chords, and—still better—the measured lengths of several, may be employed accurately to determine the instant the Sun's centre would transit each particular bar.

Let C be the Sun's centre approaching the brilliant bar BB′ at the precise mean time M. Let BB′ be measured on the screen placed at a known fixed distance from the object-glass, so that the apparent diameter of the Sun may be assignable on the screen; so as to compare the length of the bar BB′ with the diameter of the Sun.

Then the time which will elapse before the centre of the Sun will transit over BB′ is (r being the semi-diameter of the Sun in time, θ the angle BCA, and $\frac{1}{2}\Delta$ being half the length of the bar)

$$t = \mathrm{CN} = r \cos \theta.$$

Also

$$\sin \theta = \frac{\mathrm{BN}}{\mathrm{BC}} = \frac{\frac{1}{2}\Delta}{r}.$$

Then the time of transit of Sun's centre over BB′ will be $\mathrm{M}+t$.

If several bars be measured on the photograph, say three

$$\mathrm{T, the\ mean\ time,} = \frac{1}{3}(t_1 + t_2 + t_3),$$

or

$$\mathrm{T} = \frac{r}{3} \cdot (\cos \theta_1 + \cos \theta_2 + \cos \theta_3);$$

and the final result for the time of passage of Sun's centre is $\mathrm{M}+\mathrm{T}$.

Should some of the bars be to the West and the others to the East, cos θ will have contrary signs algebraically.

Or if there are n bars measured, then, according to the usual notation,

$$\mathrm{T} = \frac{r}{n} \Sigma (\cos \theta).$$

Now, admitting that the lines are ruled very accurately (a thing not approached by mechanical arrangement of spider lines), any one of these lines should give the same instant of transit, because the centre passes each after equal intervals. By inspecting the tables of natural sines and tangents, it appears that, up to the first 18 minutes of arc, the tangents and sines are identical up to the $\frac{1}{10{,}000{,}000}$th of the radius, and the minutes of arc up to the semi-diameter of the Sun are equal within the

$\frac{1}{10,000,000}$th of the radius. From this it follows that the measurements of the bars, if drawn equidistant, for a telescope of 100 inches focus will not exhibit a tangential error exceeding the $\frac{1}{10,000}$th of an inch. At the full breadth of the image of the Sun—32 minutes—the error would be no greater. But as the field of view would probably not much exceed the semi-diameter, the tangential error from employing equidistant lines may be neglected.

The difficulty of ascertaining on the screen the precise magnitude of the Sun's semi-diameter may be avoided by using a screen at a standard distance from the focal point of the object-glass. And if a single uncemented triplet be used to throw the image upon the screen, whose focal length for simplicity is 1 inch, say at a distance of 10 inches from it, the usual formulæ for optical foci will give

$$\frac{1}{10}+\frac{1}{v}=\frac{1}{1}, \quad \text{or} \quad v=\frac{10}{9},$$

and the magnifying power on the screen will be

$$\frac{u}{v}=10\div\frac{10}{9}=9,$$

so that, at the distance of 10 inches from the eye-lens of the telescope (with a 1-inch triplet), the disk of the Sun will be magnified exactly *nine* times.

The actual size of the disk of the Sun at the focus of the telescope is readily obtained from the equation

$$D = F \sin \delta,$$

F being its focal length and δ the diameter of the Sun.

If the focal length be 60 inches, for example, and the Sun's diameter be $32' 20''{\cdot}2$, the diameter on the screen will be

$$\delta = 9F \sin 32' 20''{\cdot}2$$

$$\log \delta = \begin{cases} \log 9 & = 0{\cdot}95424 \\ +\log 60 & = 1{\cdot}77815 \\ +\log 32' 20''{\cdot}2 & = 7{\cdot}97334 \end{cases}$$

(rejecting the index) $10{\cdot}70573 = 5{\cdot}078$ inches

By a slight shifting of the screen, the Sun's image for the day may be made exactly to correspond with 5 inches. Should that be too large a photograph, then a telescope of shorter length can be used. At 6 inches distance the image will be magnified five

times, just one time less than the number of inches of the screen from the lens, as seen from the formula

$$\frac{1}{d}+\frac{1}{v}=1,$$

for then

$$v=\frac{d}{d-1},$$

and the diameter of the Sun will be 2·821 inches. At 3 inches it will be magnified twice, and show a disk of diameter 1·128 inches. This very useful law for a 1-inch lens gives at once the magnifying power at any distance; say 30 inches distance from the lens, the image will be 29 times greater than the object in its focus.

Doubtless many methods will suggest themselves for utilising the appearance of the brilliant solar bars during observations for a transit. Suppose the photograph taken shortly before the time of central transit, so that the whole system of equidistant parallel bars are illuminated: the most favourable bar will evidently give the most exact position of the Sun's centre and the time of transit, viz. the bar or bars which give the smallest chords; for here they increase most rapidly.

Should the observer calculate the times from several bars, and take the mean of each, I presume this mean would scarcely differ from that obtained from the most favourable bar.

In photographing, the effect of obliquity of vision or parallax by displacement of the eye from the true optical axis is avoided. Also another error, irradiation, will so nearly affect both the disk of the Sun and the extremities of the brilliant bars as to lessen this source of error, which for *Venus* is about, I believe, half a second, and probably not much more for the Sun. Now half a second will not be much on the screen, and at each end of the bar this will make a whole second; and on a screen placed at 9 inches, this will with the telescope in question be one four-hundredth of an inch, and for a disk 2 inches in diameter it will be reduced to a thousandth. I do not therefore apprehend irradiation will be a serious error, if neglected. But it will be measurable by comparing a maximum bar with the Sun's calculated diameter on the screen.

In the case where a second photograph is taken after central transit, there will probably be great choice of verification bars. This method does not necessarily require the Sun-picture to be taken at the instant of transit.

But if such a system were preferred, a thin plate of glass placed at right angles to the axis, between the lens and the screen, and close behind the former, would both transmit the picture and reflect it to the observer in parallel rays by a suitable lens. The observer could then, by a spring, flash the photograph at the most favourable instant.

Supposing several bars are then chosen, avoiding those which

are formed nearly equal to the diameter of the Sun, the determination of the distance of the Sun's centre from each of the chosen bars will be much facilitated by using the natural cosines of θ after determining the value of θ from the equation

$$\sin\theta = \frac{\frac{1}{2}\Delta}{r} = \frac{\text{half a bar}}{\text{rad. of Sun}}.$$

The foregoing equation

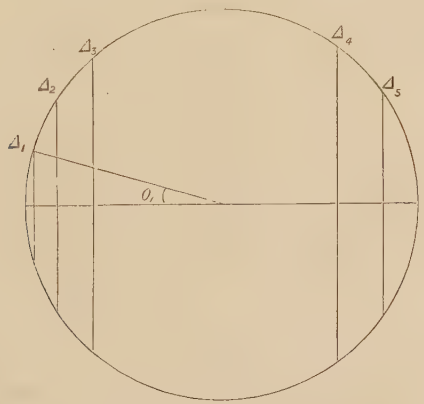
$$\mathrm{T} = \frac{r}{n}\,\Sigma\,(\cos\theta)$$

will then give the time of transit of the Sun's centre, and the sign of the result, whether positive or negative, will show whether the time of observation is before or after the true transit. Moreover, should the first numerical differences not be exactly constant between $\cos\theta_1$, $\cos\theta_2$, $\cos\theta_3$, &c., two things may happen: either the lines are not ruled precisely equidistant, or else the measurement of the bars is untrue. For, since $t = r\cos\theta$, and r is constant, t varies as $\cos\theta$, so that $\cos\theta$ must increase exactly *pari passu* with the time the Sun has passed, or will reach a given bar.

After taking the sum of the natural cosines arithmetically, the final value of $\pm\mathrm{T} = \frac{r}{n} \times$ sum of the cosines will of course be readily found by logarithms.

Probably sufficient accuracy (as perhaps diminishing the effect of irradiation) will be attained by preferring bars forming chords nearly subtending 60°, or equal to the Sun's radius.

A trigonometrical example may possibly interest some of our Fellows, and I therefore beg to submit one.



Let the bars

Δ_1	Δ_2	Δ_3	Δ_4	Δ_5
+18·2	+25·6	40·2	40	32·2

be measured; determine the value of the times of transit of the Sun's centre over each bar: ($r = 32{\cdot}68$)

$$\sin\theta_1 = \Delta_1 \div 2r = \frac{18{\cdot}2}{2\times 32{\cdot}68} = \frac{9{\cdot}1}{32{\cdot}68}$$

$$\log\sin\theta_1 = \left\{\begin{matrix} \cdot 9590 \\ -1{\cdot}5143 \end{matrix}\right\} = 9{\cdot}4447,\ \theta_1 = 16^\circ\ 10'$$

$$\log\cos\theta_1 = 9{\cdot}9825$$

$$\log r\cos\theta_1 = \left\{\begin{matrix} 1{\cdot}5143 \\ +9{\cdot}9825 \end{matrix}\right\} = 1{\cdot}4968,\ t_1 = \underline{31^s{\cdot}39}$$

$$\sin\theta_2 = \frac{12{\cdot}8}{32{\cdot}68},\ \log\sin\theta_2 = \left\{\begin{matrix} 1{\cdot}10721 \\ -1{\cdot}51428 \end{matrix}\right\} = 9{\cdot}59293,\ \theta_2 = \sin 23^\circ\ 4'$$

$$\left.\begin{matrix} \log\cos\theta_2 = & 9{\cdot}9638 \\ +\log r = & +1{\cdot}5143 \end{matrix}\right\} = 11{\cdot}4781,\ t_2 = \underline{30^s{\cdot}07}$$

$$\sin\theta_3 = \frac{20{\cdot}1}{32{\cdot}68},\ \log\sin\theta_3 = \left\{\begin{matrix} 1{\cdot}3031 \\ -1{\cdot}5143 \end{matrix}\right\} = 9{\cdot}7888,\ \theta_3 = 37^\circ\ 57'$$

$$t_3 = r\cos\theta_3 \left\{\begin{matrix} \log\cos\theta_3 = & 9{\cdot}8968 \\ +\log r = & +1{\cdot}5143 \end{matrix}\right\} = 1{\cdot}4111,\ t_3 = \underline{25^s{\cdot}77}$$

$$\sin\theta_4 = \frac{20}{32{\cdot}68},\ \log\sin\theta_4 = \left\{\begin{matrix} 1{\cdot}3010 \\ -1{\cdot}5143 \end{matrix}\right\} = 9{\cdot}7867,\ \theta_4 = 37^\circ\ 44'$$

$$t_4 = r\cos\theta_4 \left\{\begin{matrix} \log\cos\theta_4 = & 9{\cdot}8981 \\ +\log r = & +1{\cdot}5143 \end{matrix}\right\} = 11{\cdot}4124,\ t_4 = \underline{25^s{\cdot}85}$$

$$\sin\theta_5 = \frac{16{\cdot}1}{32{\cdot}68},\ \log\sin\theta_5 = \left\{\begin{matrix} 1{\cdot}2068 \\ -1{\cdot}5143 \end{matrix}\right\} = 9{\cdot}6925,\ \theta_5 = 29^\circ\ 31'$$

$$\left.\begin{matrix} \log\cos\theta_5 = & 9{\cdot}9396 \\ \log r = & +1{\cdot}5143 \end{matrix}\right\} = 1{\cdot}4539,\ t_5 = \underline{28^s{\cdot}44}$$

The bars Δ_1, Δ_2, Δ_3 lie east of Sun's centre and are negative. Times of Sun's centre passing the

Bars	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	if M be M.T. of transit.
M ±	$-31^s{\cdot}39$	$-30^s{\cdot}07$	$-25^s{\cdot}77$	$25{\cdot}85$	$28{\cdot}45$	
Differences	$1^s{\cdot}32$	$4^s{\cdot}30$	$51^s{\cdot}62$	$2^s{\cdot}60$		

By the microscope the intervals of each can be accurately measured with ease to the 10,000th of an inch, as also the width of the cuts. By a low power, the photograph would probably yield the constant of irradiation for the given telescope, which, however, will probably vary with greater or less brilliancy of the Sun, and states of the atmosphere.

The extremity of each bar will be projected outwards by the irradiation; while the breadth of a bar will be determined by mixed optical causes, such as diffraction and interference, as the peculiar darkness of the bright bands before illumination by the Sun particularly denotes. Again, the known diameter of the Sun compared with its image will be another method for determining readily this error.

I offer this method with great diffidence to the Society, having myself no practical knowledge of photography. Nor am I able to offer any valuable opinion whether this method of silver-film bars for transits is of any advantage. For myself, I have been contented with watching the very beautiful appearances of the transit of the Sun across these film bars, either projected on a screen or directly with a darkened eye-piece.

The method of direct observation by projection appears to have some advantages. The screen can readily be sketched with an exact copy of the illuminated bars, and divided into a scale of parts; the observer, at a given tick of the clock, can mark with a pencil the precise position of the extremity of a given bar. The diameter of the Sun may easily be enlarged to a foot. The lens spoken of as a triplet is an extraordinarily fine one, by Wray, formed of three glasses cemented to each other with balsam, and capable of displaying a very fine image of the Sun at 10 inches.

Should the solar heat melt the balsam, Mr. Wray can very readily construct these triplets without balsam. And if one of the interior surfaces is faintly blued with silver, the glass becomes a useful and beautifully defining eye-piece, though of small field. A large shade of pasteboard slipped over the object end of the telescope greatly improves the splendour and distinctness of the projected image of the Sun; and a number of transits across the bars, by the commencing flash and length, can be taken with great ease and comfort by the observer, the eye being entirely saved the heat, glare, and strain usual to the generally adopted method of spider lines. But, I again beg to repeat, I feel quite incompetent to form any opinion of the real value or utility of the methods now described.

I would add (while correcting the proof) that if the silver film be just the sufficient thickness or opacity to permit the outline of the limb of the Sun to be photographed, it is probable that nearly the whole of the irradiation of the limb may thus be destroyed, leaving that of the extremities of the bars in full irradiatory display.

The time can also afterwards, by the same translucent bars be compared with a star transit.

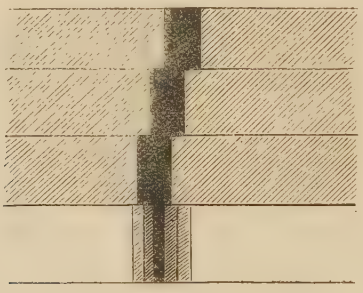
Effect of a Star's Rotation on its Spectrum.

By Capt. W. de W. Abney.

Having privately asked our President at the last meeting if he had considered the effect that would be produced by the rotation of a star on its spectrum, and having received a negative answer, I have ventured to bring the subject before the Society to provoke discussion if possible.

The light which radiates from every part of the visible surface of the star will fall on the slit of the spectroscope; hence a separate spectrum due to every portion of it will be formed. The advancing limb—supposing there is a rotation of the star—will cause the absorption lines to move towards the violet end, and the receding limb towards the red end of the spectrum; whilst the central portion will cause them to occupy their normal positions, provided there be no motion of the star in space.

As an example of what would occur, let us suppose the star's disk to be divided into three equal parts; one of which is advancing towards us, another receding at the same rate, and the third motionless. If we draw an exaggerated diagram of a small part of the spectra given by the light radiating from these three surfaces, we should get a result as shown.



There would be a total broadening of the line, consisting of a sort of double penumbra and a black nucleus. If the displacement was equal to half of the breadth of the line, the double penumbra alone would remain; whilst, if it were equal to the total breadth of the line, there would be only one penumbra. If we suppose the stellar surface to be equally bright throughout, we should get a graduated shade forming the penumbra; which would gradually melt off into the blacker nucleus which would form the line. I have calculated what the shade would be; but I have not thought it necessary to bring forward the formulæ to-night.

It seems, supposing the surface of a quickly rotating star

to be in the same condition as the Sun as regards gaseous pressure and temperature, that no absorption line due to the former can be as sharp and distinct as it is when seen in the solar spectrum. More than this, rotation might account for the disappearance of some of the finer lines of the spectrum ; and perhaps other phenomena observed may be attributable to the same cause. It appears also that, other conditions being known, the mean velocity of rotation might be calculated, or perhaps even the angular velocity. Since Dr. Huggins has commenced taking photographs of the spectra of stars, he may be able to give a thought to these points. I am convinced that from a good photograph much might be determined.

On the Variable Star U Geminorum. By G. Knott, Esq.

Mr. Hind's curious variable star, U *Geminorum*, has recently passed a *maximum*, of which I have obtained the following pretty complete series of observations :—

			h	m		Mag.
1877	February	16	11	40	G.M.T.	14·0
		17	7	30		14·0
		20	8	10		13·2
		21	10	30		9·8
		23	8	45		9·9
		25	10	0		9·9
		26	9	0		9·85
		27	9	0		9·9
		28	7	0		9·9
	March	1	7	35		9·95
		4	11	50		11·2
		5	10	30		11·7
		6	9	0		12·3
		8	8	0		13·7

The maximum was a rather faint one. The colour of the star throughout its apparition was white or bluish white, and on February 27 and March 4 was noted as *hazy*.

Cuckfield.
1877, *March* 9.

Note on the Double Star ♅ I. 62. By S. W. Burnham, Esq.

The place of No. 62 of Sir William Herschel's Class I. is given in the Synopsis of his Catalogues (*Memoirs of the R.A.S.*, vol. xxxv.) for 1880, as:

R.A. $= 20^h\ 58^m{\cdot}8$.
Decl. $= +6°\ 17'{\cdot}6$.

No measures of distance are recorded; but the angle was measured twice, 234°·8 in 1783·4, and 237°·1 in 1802·8. In the course of a search for the double stars of Herschel's Class I., not found in *Mensurae Micrometricae*, this was carefully looked for several times in and near the assigned place, but nothing of the kind could be found; and I became satisfied that in this, as in many other pairs previously identified, there was a considerable error in the place. It has since occurred to me that this is probably identical with No. 269 of my Fifth Catalogue of New Double Stars (*Monthly Notices*, November 1874), there being an error of 1° in Herschel's declination. The place of the new pair (1880) is:—

R.A. $= 20^h\ 58^m\ 39^s$.
Decl. $= +7°\ 19'\ 6''$.

Baron Dembowski has measured this on three nights, giving a mean result (1875·1), $P = 251°{\cdot}0$; $D = 1''{\cdot}11$. The direct motion indicated by the two observations of Herschel would make the position-angle about 245° in 1875. As the pair is somewhat difficult, a small error in either of the single measures would fully account for this slight discrepancy. As similar mistakes in declination are not unfrequent in the lists of both Herschels, it may be considered as almost beyond doubt that these two pairs are identical. A continued advance in the angle will settle the question. There remains but one pair not yet accounted for of Herschel's Class I.

Chicago,
1877, *January* 23.

On the Magnitude of η Argûs. By J. Tebbutt, Esq.

In the *Monthly Notices*, vol. xxxi., p. 211, was published a series of comparisons of *η Argûs* made by me, and extending from July 1854 to the end of 1870. The following results are to be regarded as a continuation of that series, the comparison stars being contained in the Catalogue on page 42 of Sir J. Herschel's *Results of Astronomical Observations at the Cape of Good Hope*,

1834–8, and embraced in the same field of view with η *Argûs*. It appears that η *Argûs* has diminished only about half a magnitude since 1870.

	Dates of the Comparisons.	Mean Date of Comparisons.	Mean of Concluded Mags.
1871	Jan. 14, 25, Feb. 13, March 20, April 3, 22, May 3, 16, 31, June 29	1871·26	6·37
1871	July 13, 14, 28, Aug. 31, Oct. 16, Nov. 25, Dec. 13, 29	1871·74	6·44
1872	Jan. 16, Feb. 20, March 8, April 5, May 9, 20, June 21, July 23, Aug. 20, Dec. 10	1872·40	6·49
1873	Feb. 4, 18, March 21, April 7, 22, 23, 24, May 1, 26, June 5, July 12, Aug. 5	1873·32	6·55
1874	Jan. 16, Feb. 18, March 19, April 17, May 13, 15, 21, 23, June 13, July 16, Nov. 19, Dec. 28	1874·42	6·58
1875	Jan. 28, March 19, May 13, 22, July 5, Aug. 17, Nov. 30, Dec. 18	1875·50	6·70
1876	June 17, 24, March 13, May 11, July 3, Aug. 31, Sept. 18, Nov. 21, Dec. 28	1876·44	6·70

Windsor, N. S. Wales,
1877, *January* 10.

On the Nomenclature of Meteor Radiant Points.

By J. W. Backhouse.

The proper naming of meteor showers is a subject of some importance, on account of the confusion that must result if unsuitable names are given to them. It therefore appears to me that it should not be left to individual astronomers to give names, or we shall have such ones as are found in the January Number of the *Monthly Notices*, pp. 108–111. In that paper there are several cases in which two different showers are called by the same name; and we find such names as *Taurids I.*, *Taurids II.*, *Leonids Minor*, &c.

I would suggest that the nomenclature of radiant points should be placed in charge of the British Association Committee on Luminous Meteors; though I do not know what foreign astronomers would say to this, seeing that they might be inclined to adopt the English names, or similar ones, on condition of their being suitably chosen. It is probably not needful that many showers should be named; but it seems most suitable that whoever wishes to have any named should communicate with that Committee.

On the Relative Space-penetrating Power of Reflectors and Refractors. By the Rev. T. R. Robinson, D.D.

I have at last obtained access to the Memoir in the *Bulletin de l'Académie de St.-Pétersbourg* to which M. Struve refers me.

It shows that M. Radau did not make the mistake which I imputed to him. I may be excused for having done so, because I did not conceive it possible that one whose authority is so high as M. O. Struve's deservedly is could assert that a 4-feet reflector of first rate excellence had scarcely higher power than a 15-inch refractor.

I was proceeding to show that the observations recorded in this Memoir lead to a conclusion the very reverse of M. Struve's, when I saw in the *Monthly Notices* that Mr. Lassell has so completely disposed of the question that such examination of them is quite unnecessary.

But I will give some facts relative to another 4-feet, the Great Melbourne Telescope, which throw some light on the question. The American astronomers who observed the Transit of *Venus* visited Melbourne, and report of that instrument to Professor Newcomb that "its definition is good; its optical power comparable to the 26-inch refractor; sky is black close to large stars instead of the blue light of the secondary spectrum shown by the great refractor; and that the distant companion of *Procyon* could have been seen close up to *Procyon* itself."

It should be mentioned that this speculum has not been repolished for several years, though it has been in constant use.

Armagh,
1877, *February* 8.

ERRATUM.

Vol. xxxvi. p. 361, middle:

for $-1''\cdot50$ and $-1''\cdot50$ read in each case $+1''\cdot50$.

viz. the correction to the longitude is

$$\delta v = +1''\cdot50 \sin (g + \mathrm{N} - 90^\circ).$$